Labor Markets

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While the real business cycle model can successfully account for the dynamics of macroeconomic aggregates such as output, consumption and investment, the assumption of perfectly competitive labor markets is not consistent with key empirical facts characterizing the labor market. For instance, in the standard RBC model all of the cyclical variation in hours worked is due to a (highly) elastic response by employed workers to movements in the real wage. We saw before that in the data, almost all of the variation is instead accounted for by changes in employment, and real wages appear to be acyclical. Moreover, the frictionless labor market does not feature involuntary unemployment: there is no worker who would like to work at the market wage but is unable to. The presence of involuntary unemployment obviously has important implications for welfare and macroeconomic policy.

One of the explanations for the existence of equilibrium unemployment is the presence of search frictions in labor market. Early examples of RBC models that incorporate labor market search are Merz (1995), Andolfatto (1996). The material below is similar to Shimer (2010). The primary goal of this chapter is to verify whether labor market search permits a model of business cycles driven by productivity shocks in which total hours worked is strongly procyclical, not because of variation in hours per worker but because of variation in employment that is partly involuntary.

1 A Simple Search Model without Capital

Households There is a measure one continuum of individuals indexed by $i \in [0, 1]$. Each individual is either employed and spends its unit time endowment working $L_t^i = 0$, or unemployed in which case $L_t^i = 1$. Everyone has the same instantaneous utility function $\log C_t^i - \gamma(1 - L_t^i)$ where C_t^i denotes consumption of individual i, and discounts future utility flows by $0 < \beta < 1$. Because individuals experience different labor market histories, consumption choices depend on the extent to which agents can insure against idiosyncratic labor income risk. For tractability, we will implicitly assume insurance is perfect (complete markets). Suppose that the individuals belong to a representative household that allocates total consumption C_t to maximize the sum of utilities. This amounts to equalizing the marginal utility of consumption across individuals. Because preferences are additively separable in consumption and leisure, this implies $C_t = C_t^i$. The large household is thus able to fully insure against idiosyncratic labor income risk and everyone enjoys the same level of consumption. The representative household's total welfare can than be formulated as function of stochastic sequences of $C_t \geq 0$ and $0 \leq N_t \leq 1$, the fraction of employed individuals:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \gamma N_t\right) \ , \quad 0 < \beta < 1 \ , \ \gamma > 0 \tag{1}$$

The flow budget constraint is

$$C_t + q_t S_{t+1} \leq (q_t + d_t) S_t + w_t N_t \ , \ S_0 \text{ given} \tag{2}$$

where S_t denotes the household's equity holdings, q_t is the price of equity and d_t are dividends.

The household is constrained by the flows of its members in and out of employment, over which it has no direct control. A time invariant random fraction ξ of workers become unemployed each period, such that ξ is the *employment-exit probability.* A random fraction ϕ_t of unemployed workers find a job. The process for the *job finding probability* ϕ_t is taken as given by the representative household. The law of motion for employment is therefore for

$$N_t = (1-\xi)N_{t-1} + \phi_{t-1}(1 - N_{t-1}) \ \ , N_{-1}, \phi_{-1} \text{ given} \tag{3}$$

The household problem is to choose the contingent sequences for $\{C_t, N_t, S_{t+1}\}_{t=0}^{\infty}$ to maximize (1) subject to the budget constraints (2) and the law of motion for employment (3) and given prices $\{r_t^a, w_t\}_{t=0}^{\infty}$ and $\{\phi_t\}_{t=0}^{\infty}$.

The optimality conditions at an interior solution include

$$\lambda_t = 1/C_t \tag{4}$$

$$q_t \lambda_t = \beta E_t \left[\lambda_{t+1} \left(q_{t+1} + d_{t+1}\right)\right] \tag{5}$$

$$V_t^n = -\gamma + \lambda_t w_t + \beta(1 - \xi - \phi_t) E_t V_{t+1}^n \tag{6}$$

where $V_t^n \geq 0$ is the marginal value of an additional job to the household.

Firms There is a large number of identical firms. The representative firm produces a final consumption good using a linear technology

$$Y_t = A_t N_t^y \tag{7}$$

where $N_t^y \geq 0$ are workers employed in the production of the consumption good and A_t is a stochastic level of productivity. The firm also employs $N_t^r \geq 0$ individuals as recruiters. Let $\mu_t \geq 0$ denote the number of job matches created per recruiter in period t leading to new hires. The process for productivity in hiring μ_t is taken as given by the representative firm. The law of motion for the firm's employment is therefore for $t > 0$

$$N_t^y + N_t^r = (1-\xi)(N_{t-1}^y + N_{t-1}^r) + \mu_{t-1} N_{t-1}^r \quad , N_{-1}^y, N_{-1}^r, \mu_{-1} \text{ given} \tag{8}$$

The firms objective is to choose $\{N_t^y, N_t^r\}_{t=0}^\infty$ to maximize its value to the shareholders $J \equiv \lambda_0(q_0 + d_0)S_0$, which is equivalent to maximizing the net present value (in utils) of dividend payments $d_t = A_t N_t^y - w_t(N_t^y + N_t^r)$, or

$$J = E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left(A_t N_t^y - w_t(N_t^y + N_t^r) \right) \tag{9}$$

subject to (8) and $N_t^y, N_t^r \geq 0$ and taking as given $\{A_t, \lambda_t, w_t, \mu_t\}_{t=0}^\infty$. The optimality conditions at an interior solution include

$$J_t^n = \lambda_t(A_t - w_t) + \beta(1-\xi) E_t J_{t+1}^n \tag{10}$$

$$J_t^n = -\lambda_t w_t + \beta(1 - \xi + \mu_t) E_t J_{t+1}^n \tag{11}$$

where $J_t^n \geq 0$ is the marginal value of an additional job to the firm. Combining yields

$$\lambda_t A_t = \mu_t \beta E_t J_{t+1}^n \tag{12}$$

The firm allocates recruiters such that the current cost in terms of foregone production A_t equals the benefit of higher expected future profits. Substituting into (11),

$$J_t^n = \lambda_t \left(\left(1 + \frac{1-\xi}{\mu_t} \right) A_t - w_t \right) \tag{13}$$

Hiring an additional employee has the direct benefit of increasing production by A_t and the additional future benefit of freeing up $\frac{1-\xi}{\mu_t}$ recruiters to become production workers. The value of a job to the firm is this benefit net of the real wage.

Equilibrium There are still several missing pieces before we can describe an equilibrium in the environment above. Households and firms treat job finding probabilities ϕ_t and the hiring rates μ_t as given, and we still need to specify how these are determined. It seems reasonable to assume that the hiring rate of recruiters depends on the number job searchers (the aggregate unemployment level $1 - N_t$) as well as the aggregate recruiting intensity of firms (the measure of recruiters N_t^r). More specifically, suppose that

$$\mu_t = \mu(\theta_t) \quad , \quad \theta_t = \frac{N_t^r}{1 - N_t} \tag{14}$$

where θ_t is the recruiter-unemployment ratio in the economy. The function $\mu(\cdot)$ is continuous, $\mu'(\cdot) \leq 0$ and $\lim_{\theta \to 0} \mu(\theta) = \infty$ and $\lim_{\theta \to \infty} \mu(\theta) = 0$. The total measure of unemployed individuals hired by firms in period t is $N_t^r \mu(\theta_t)$, and therefore to balance the employment flows it must be the case that

$$\phi_t \equiv \phi(\theta_t) = \theta_t \mu(\theta_t) \tag{15}$$

where $\phi(\cdot)$ is nondecreasing, concave with $\phi(0) = 0$ and $\phi(\cdot) \leq 1$.

The model also still lacks a mechanism for pinning down real wages, because once a worker is matched with an employer there generally is a range of values for the real wage for which individuals are willing to work and firms are willing to employ the worker. To see this, note that the value of an additional job to the firm J_t^n given by (13) is nonnegative for any w not exceeding the marginal benefit to the firm:

$$w \leq \left(1 + \frac{1 - \xi}{\mu_t}\right) A_t$$

Similarly, the value of an additional job to the household V_t^n given by (6) is positive as long as

$$w \geq \gamma/\lambda_t - \beta(1 - \xi - \phi_t) E_t V_{t+1}^n / \lambda_t$$

or whenever the after-tax real wage exceeds the marginal rate of substitution between consumption and leisure γ/λ_t minus the expected benefit of working due to the fact that current employment increases the probability of being employed in the future. For any wage satisfying both inequalities, the employment relationship is mutually beneficial.

One approach is to view the wage as arising from a bargaining process between the firm and the worker. A common assumption is that such bargaining takes places at the beginning of every period and that it results in the worker and the firm splitting the total gains from trade in constant proportions. Given that the total before-tax surplus is $J_t^n + V_t^n$, the bargaining outcome is assumed to result in

$$\begin{aligned} V_t^n &= \zeta\left(J_t^n + V_t^n\right) && (16) \\ J_t^n &= (1-\zeta)\left(J_t^n + V_t^n\right) && (17) \end{aligned}$$

where $0 \leq \zeta \leq 1$ parametrizes the bargaining weight of the worker. Such an outcome can be motivated as a Nash bargaining solution.

Finally, in equilibrium markets must clear:

- Labor Market: $N_t = N_t^r + N_t^y$
- Asset Market: $S_t = 1$
- Goods Market: $C_t = Y_t$

Consolidating the optimality and market clearing conditions and adding the Nash bargaining assumption, if the stochastic sequences $\{Y_t, N_t, V_t^n, J_t^n, w_t, \theta_t\}_{t=0}^{\infty}$ with $0 < N_t, \theta_t < 1$ solve

$$\begin{aligned} Y_t &= (1+\theta_t)A_t N_t - A_t\theta_t && (18) \\ N_{t+1} &= (1-\xi)N_t + \phi(\theta_t)(1-N_t) && (19) \\ J_t^n &= \frac{1}{Y_t}\left(\left(1 + \frac{1-\xi}{\phi(\theta_t)}\theta_t\right)A_t - w_t\right) && (20) \\ A_t\theta_t/Y_t &= \phi(\theta_t)\beta E_t J_{t+1}^n && (21) \\ V_t^n &= -\gamma + \frac{w_t}{Y_t} + \beta(1-\xi-\phi(\theta_t))E_t V_{t+1}^n && (22) \\ \zeta J_t^n &= (1-\zeta)V_t^n && (23) \end{aligned}$$

given an exogenous stochastic process $\{A_t\}_{t=0}^{\infty}$ and given an initial employment level N_0, they constitute an equilibrium.

Define $y_t = Y_t/A_t$ and $\tilde{w}_t = w_t/A_t$, and rewrite the system as

$$\begin{aligned}
y_t &= (1+\theta_t)N_t - \theta_t && (24)\\
N_{t+1} &= (1-\xi)N_t + \phi(\theta_t)(1-N_t) && (25)\\
J_t^n &= \frac{1}{y_t}\left(\left(1+\frac{1-\xi}{\phi(\theta_t)}\theta_t\right) - \tilde{w}_t\right) && (26)\\
\theta_t/Y_t^s &= \phi(\theta_t)\beta E_t J_{t+1}^n && (27)\\
V_t^n &= -\gamma + \frac{\tilde{w}_t}{y_t} + \beta(1-\xi-\phi(\theta_t))E_t V_{t+1}^n && (28)\\
\zeta J_t^n &= (1-\zeta)V_t^n && (29)
\end{aligned}$$

Note that this new system is independent of the stochastic process A_t and is solved by a deterministic sequence for $\{y_t, N_t, V_t^n, J_t^n, \tilde{w}_t, \theta_t\}_{t=0}^{\infty}$. In other words, employment dynamics are independent of A_t and technology shocks are neutral for (un)employment!

Provided the system is saddle-path stable, for an initial employment level N_0 the system will converge to a steady state

$$\begin{aligned}
\bar{N} &= \phi(\bar{\theta})/(\xi+\phi(\bar{\theta})) && (30)\\
\bar{y} &= \frac{\phi(\bar{\theta})-\xi\bar{\theta}}{\xi+\phi(\bar{\theta})} && (31)\\
\bar{J}^n &= \frac{\bar{\theta}}{\beta\phi(\bar{\theta})}\left(\frac{\xi+\phi(\bar{\theta})}{\phi(\bar{\theta})-\xi\bar{\theta}}\right) && (32)\\
\bar{V}^n &= \frac{-\gamma+\left(1-\frac{1-\beta(1-\xi)}{\phi(\bar{\theta})\beta}\bar{\theta}\right)\left(\frac{\xi+\phi(\bar{\theta})}{\phi(\bar{\theta})-\xi\bar{\theta}}\right)}{1-\beta(1-\xi-\phi(\bar{\theta}))} && (33)\\
\bar{\tilde{w}} &= 1-\frac{1-\beta(1-\xi)}{\phi(\bar{\theta})\beta}\bar{\theta} && (34)\\
& && (35)
\end{aligned}$$

where $\bar{\theta}$ is implicitly determined by

$$\zeta\bar{J}^n = (1-\zeta)\bar{V}^n \qquad (36)$$

At the steady state employment level, output and wage dynamics are given by

$$\begin{aligned}
Y_t &= A_t\left((1+\bar{\theta})\bar{N}-\bar{\theta}\right) && (37)\\
w_t &= A_t\left(1-\frac{1-\beta(1-\xi)}{\phi(\bar{\theta})\beta}\bar{\theta}\right) && (38)
\end{aligned}$$

To gain some intuition for this neutrality result, combine (20)-(22) to obtain

$$w_t \quad = \quad (1-\zeta)\gamma Y_t + \zeta(1+\theta_t)A_t \tag{39}$$

This equation states that the wage paid is a weighted average between γY_t, or the equilibrium marginal rate of substitution between consumption and leisure, and $(1+\theta_t)A_t$, or the marginal product of labor, with constant weights determined by the bargaining parameter ζ. Because of log utility and the linear production technology, both terms are proportional to A_t, and hence the real wage is also proportional to A_t, regardless of the value of ζ. An increase in A_t creates an incentive for firms to employ more production workers. Productivity gains result in higher wages and capital gains for the households, and therefore generate an income effect on consumption. This income effect increases the lowest wage that household require to employ the marginal worker, which raises the bargained wage to the point where firms no longer wish to expand employment. Any incentive to allocate more workers to production rather than recruiting is exactly offset by the need for additional recruiters to maintain optimal employment in the future.

Although the exact neutrality of employment to productivity shocks relies strongly on the particular specifications of preferences and technology, it is symptomatic for the general difficulty of labor market search models to generate realistic cyclical fluctuations in employment, see Shimer (2005) and Shimer (2010). Although search frictions in the labor market can rationalize the existence of involuntary unemployment in the long run, explaining the cyclical volatility of unemployment turns does not follow automatically.

2 An RBC Model with Labor Market Search

This section extends the model above to include capital accumulation. The main objective (besides greater realism) is to verify the extent to which allowing consumption to be different from production alters the neutrality result. The households problem is the same as before and yields the same conditions as in (4)-(6).

The representative firm now operates the technology

$$Y_t = A_t K_t^{1-\alpha}(X_t N_t^y)^\alpha \quad , \quad 0 < \alpha < 1 \tag{40}$$

with $K_{t+1} = I_t + (1-\delta)K_t \quad , \quad 0 < \delta < 1 \ , \ K_0$ given, and

$$\begin{aligned} X_t &= X_{t-1}\gamma_x \quad , \quad X_0 \quad \text{given} \\ A_t &= \bar{A}e_t^a \\ a_t &= \rho a_{t-1} + \epsilon_t \ \ 0 \leq \rho < 1 \ , \ \epsilon_t \sim N(0, \sigma_\epsilon) \end{aligned}$$

As before, the firm employs $N_t^r \geq 0$ recruiters with $\mu_t \geq 0$ denoting the number of job matches created per recruiter in period t leading to new hires and employment evolving according to

$$N_t^y + N_t^r = (1-\xi)(N_{t-1}^y + N_{t-1}^r) + \mu_{t-1}N_{t-1}^r \quad , N_{-1}^y, N_{-1}^r, \mu_{-1} \text{ given} \tag{41}$$

The firms objective is to choose $\{K_{t+1}, N_t^y, N_t^r\}_{t=0}^\infty$ to maximize the net present value of dividend payments, or

$$J = E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left(A_t K_t^{1-\alpha}(N_t^y)^\alpha - K_{t+1} + (1-\delta)K_t - w_t(N_t^y + N_t^r) \right) \tag{42}$$

subject to (41) and $N_t^y, N_t^r \geq 0$ and taking as given $\{A_t, \lambda_t, w_t, \mu_t\}_{t=0}^{\infty}$. The optimality conditions at an interior solution include

$$J_t^n = \lambda_t(\alpha A_t(K_t/N_t^y)^{1-\alpha} - w_t) + \beta(1-\xi)E_t J_{t+1}^n \tag{43}$$

$$J_t^n = -\lambda_t w_t + \beta(1-\xi+\mu_t)E_t J_{t+1}^n \tag{44}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left((1-\alpha)A_{t+1}(N_{t+1}^y/K_{t+1})^{\alpha} + 1 - \delta\right) \tag{45}$$

where $J_t^n \geq 0$ is the marginal value of an additional job to the firm. Combining yields

$$\lambda_t \alpha A_t (K_t/N_t^y)^{1-\alpha} = \mu_t \beta E_t J_{t+1}^n \tag{46}$$

The firm allocates recruiters such that the current cost in terms of foregone production A_t equals the benefit of higher expected future profits. Substituting into (44),

$$J_t^n = \lambda_t \left(\left(1 + \frac{1-\xi}{\mu_t}\right)\alpha A_t (K_t/N_t^y)^{1-\alpha} - w_t\right) \tag{47}$$

As before, hiring an additional employee has the direct benefit of increasing production by $\alpha A_t(K_t/N_t^y)^{1-\alpha}$ and the additional future benefit of freeing up $\frac{1-\xi}{\mu_t}$ recruiters to become production workers. The value of a job to the firm is this benefit net of the real wage.

Bargaining between the firms and the worker results in

$$\zeta J_t^n = (1-\zeta)V_t^n \tag{48}$$

where $0 \leq \zeta \leq 1$ measures the bargaining power of the worker.

Market clearing requires

- Labor Market: $N_t = N_t^r + N_t^y$
- Asset Market: $S_t = 1$
- Goods Market: $C_t + I_t = Y_t$

Defining $y_t = Y_t/X_t$, $c_t = C_t/X_t$, $k_t = K_t/X_t$ and $\tilde{w}_t = w_t/X_t$, the equilibrium conditions yields the stationary system

$$
\begin{aligned}
y_t &= A_t k_t^{1-\alpha}\left((1+\theta_t)N_t - \theta_t\right)^{\alpha} && (49)\\
N_{t+1} &= (1-\xi)N_t + \phi(\theta_t)(1-N_t) && (50)\\
J_t^n &= \frac{1}{c_t}\left(\left(1+\frac{1-\xi}{\phi(\theta_t)}\theta_t\right)\frac{\alpha y_t}{(1+\theta_t)N_t-\theta_t} - \tilde{w}_t\right) && (51)\\
\frac{\theta_t}{c_t}\frac{\alpha y_t}{(1+\theta_t)N_t-\theta_t} &= \phi(\theta_t)\beta E_t J_{t+1}^n && (52)\\
V_t^n &= -\gamma + \frac{\tilde{w}_t}{c_t} + \beta(1-\xi-\phi(\theta_t))E_t V_{t+1}^n && (53)\\
\frac{1}{c_t} &= \frac{\beta}{\gamma_x}E_t\frac{1}{c_{t+1}}\left((1-\alpha)\frac{y_{t+1}}{k_{t+1}} + 1 - \delta\right) && (54)\\
\zeta J_t^n &= (1-\zeta)V_t^n && (55)\\
c_t + \gamma_x k_{t+1} &= y_t + (1-\delta)k_t && (56)
\end{aligned}
$$

given the exogenous stochastic process $\{A_t\}_{t=0}^{\infty}$ and initial conditions N_0, k_0.

Note that combining (51)-(53) and (55) yields the wage equation

$$
\tilde{w}_t = (1-\zeta)\gamma c_t + \zeta\left(\frac{(1+\theta_t)\,\alpha y_t}{(1+\theta_t)N_t-\theta_t}\right)
$$

As before, the (detrended) real wage is a weighted average of the marginal rate of substitution between consumption and leisure and the marginal product of labor.

Calibration As in previous chapters, we will keep on assuming $\gamma_x = 1.004$, $\beta = 0.988$, $\rho = 0.90$, $\sigma_\epsilon = 0.01$. The parameter $\bar{A}$ is chosen to normalize $\bar{y}$ to one. We will assume the following functional form for the job finding probability (also known as the *matching function*)

$$
\phi(\theta) = \min\left(\nu\theta^{\eta}, 1\right) \qquad (57)
$$

where the min operator ensures $\phi(\theta)$ is a proper probability, although the calibration below will result in $\phi(\bar{\theta}) < 1$. The remaining parameters to be calibrated are the technology parameters α and δ, the employment exit probability ξ, the bargaining weight ζ, the utility weight γ and the matching parameters ν and η. Data on the average employment exit probability in Shimer (2005) for the US averages to 0.034 per month. Since our calibration is based on a quarterly frequency, we need to compute the corresponding quarterly probability. The model assumes that employment status follows a Markov chain. The transition matrix at a *monthly* frequency is

$$P_m = \begin{bmatrix} 1-\xi_m & \xi_m \\ \phi_m & 1-\phi_m \end{bmatrix}$$

where ξ_m is the monthly employment exit probability and ϕ_m is the monthly job finding probability. The Markov transition implies that the steady state employment rate is $\bar{N} = \phi_m/(\xi_m+\phi_m)$. In US data, the average employment rate is roughly $\bar{N} = 0.95$ such that the monthly job finding probability must be $\phi_m = \xi_m\bar{N}/(1-\bar{N}) = 0.034 * 0.95/0.05 = 0.646$. The transition matrix at a *quarterly* frequency is

$$P_q = P_m^3 = \begin{bmatrix} 1-0.034 & 0.034 \\ 0.646 & 1-0.646 \end{bmatrix}^3 = \begin{bmatrix} 1-0.0484 & 0.0484 \\ 0.9189 & 1-0.9189 \end{bmatrix}$$

Therefore, the probability of entering a quarter employed but ending the quarter unemployed is $\xi = 0.0484$, whereas the probability of entering the quarter as unemployed but ending the quarter employed is $\phi(\bar{\theta}) = 0.9189$. You can verify that $\bar{N} = \phi(\bar{\theta})/(\xi+\phi(\bar{\theta})) = 0.95$. Based on business surveys, Hagedorn and Manovskii (2008) calculate that the average labor cost of hiring one worker is 3 percent to 4.5 percent of quarterly wages of a new hire. Assuming a value of 4%, this implies that a recruiter can hire 25 workers per quarter or $\mu(\bar{\theta}) = 25$. Since

$$\phi(\bar{\theta}) = \mu(\bar{\theta})\bar{\theta}$$

this implies that $\bar{\theta} = 0.9189/25 = 0.0368$. This in turn implies that the share of recruiters in total employment is $\bar{\theta} * (1-\bar{N})/\bar{N} = 0.0019$ or in other words, the implicit hiring costs are very small. We will follow Shimer (2010) and assume symmetry in bargaining, $\zeta = 0.5$, and that the elasticity of the matching function is $\eta = 0.5$. The latter then implies that $\nu = \phi(\bar{\theta})\bar{\theta}^{-\eta} = 0.9189 * 0.0368^{-0.5} = 4.79$.

Finally, the parameters α, δ and γ are chosen to match a labor income share of 0.58, and investment share in GDP of 0.295 and an employment ratio of 0.95. The values are computed numerically by solving the following system of equations for given values of the other parameters,

$$\begin{aligned}\bar{\bar{w}}\bar{N} &= \left[(1-\zeta)\gamma\bar{c}+\zeta\left(\frac{(1+\bar{\theta})\,\alpha}{(1+\bar{\theta})\bar{N}-\bar{\theta}}\right)\right]\bar{N}=0.42\\ \bar{c} &= 1+(1-\delta-\gamma_x)\bar{k}=0.705\\ \zeta\bar{J}^n &= (1-\zeta)\bar{V}^n \quad \text{for} \quad \bar{N}=0.95\end{aligned}$$

where

$$\begin{aligned}\bar{k} &= \frac{\beta/\gamma_x(1-\alpha)}{(1-\beta/\gamma_x(1-\delta)}\\ \bar{J}^n &= \frac{\bar{\theta}}{\phi(\bar{\theta})\beta}\frac{\alpha/\bar{c}}{(1+\bar{\theta})\bar{N}-\bar{\theta}}\\ \bar{V}^n &= \left(-\gamma+\frac{\bar{\bar{w}}}{\bar{c}}\right)\left(1-\beta(1-\xi-\phi(\bar{\theta}))\right)^{-1}\end{aligned}$$

Solving yields $\alpha = 0.5803$, $\delta = 0.0248$ and $\gamma = 0.8319$.

Dynamics Loglinearizing around the deterministic steady state yields

$$\hat{y}_t = \frac{1}{1-\alpha}\hat{a}_t+\hat{k}_t-\frac{\alpha}{1-\alpha}\widehat{mpl_t} \tag{58}$$

$$\widehat{mpl_t} = \hat{y}_t-\frac{\phi(\bar{\theta})(1+\bar{\theta})}{\phi(\bar{\theta})-\xi\bar{\theta}}\hat{n}_t+\frac{\xi\bar{\theta}}{\phi(\bar{\theta})-\xi\bar{\theta}}\hat{\theta}_t \tag{59}$$

$$\hat{n}_{t+1} = (1-\xi-\phi(\bar{\theta}))\hat{n}_t+\xi\eta\hat{\theta}_t \tag{60}$$

$$\hat{J}^n_t+\hat{c}_t = \beta\left(1-\xi+\frac{\phi(\bar{\theta})}{\bar{\theta}}\right)\widehat{mpl_t}+\left(1-\beta(1-\xi+\frac{\phi(\bar{\theta})}{\bar{\theta}})\right)\hat{w}_t+\beta(1-\xi)(1-\eta)\hat{\theta}_t \tag{61}$$

$$\widehat{mpl_t} = \hat{c}_t-(1-\eta)\hat{\theta}_t+E_t\hat{J}^n_{t+1} \tag{62}$$

$$\hat{V}^n_t = -\frac{1-\zeta}{\zeta}\left(1-\beta\left(1-\xi+\frac{\phi(\bar{\theta})}{\bar{\theta}}\right)\right)(\hat{w}_t-\hat{c}_t)+\beta(1-\xi-\phi(\bar{\theta}))E_t\hat{V}^n_{t+1}-\beta\phi(\bar{\theta})\eta\hat{\theta}_t \tag{63}$$

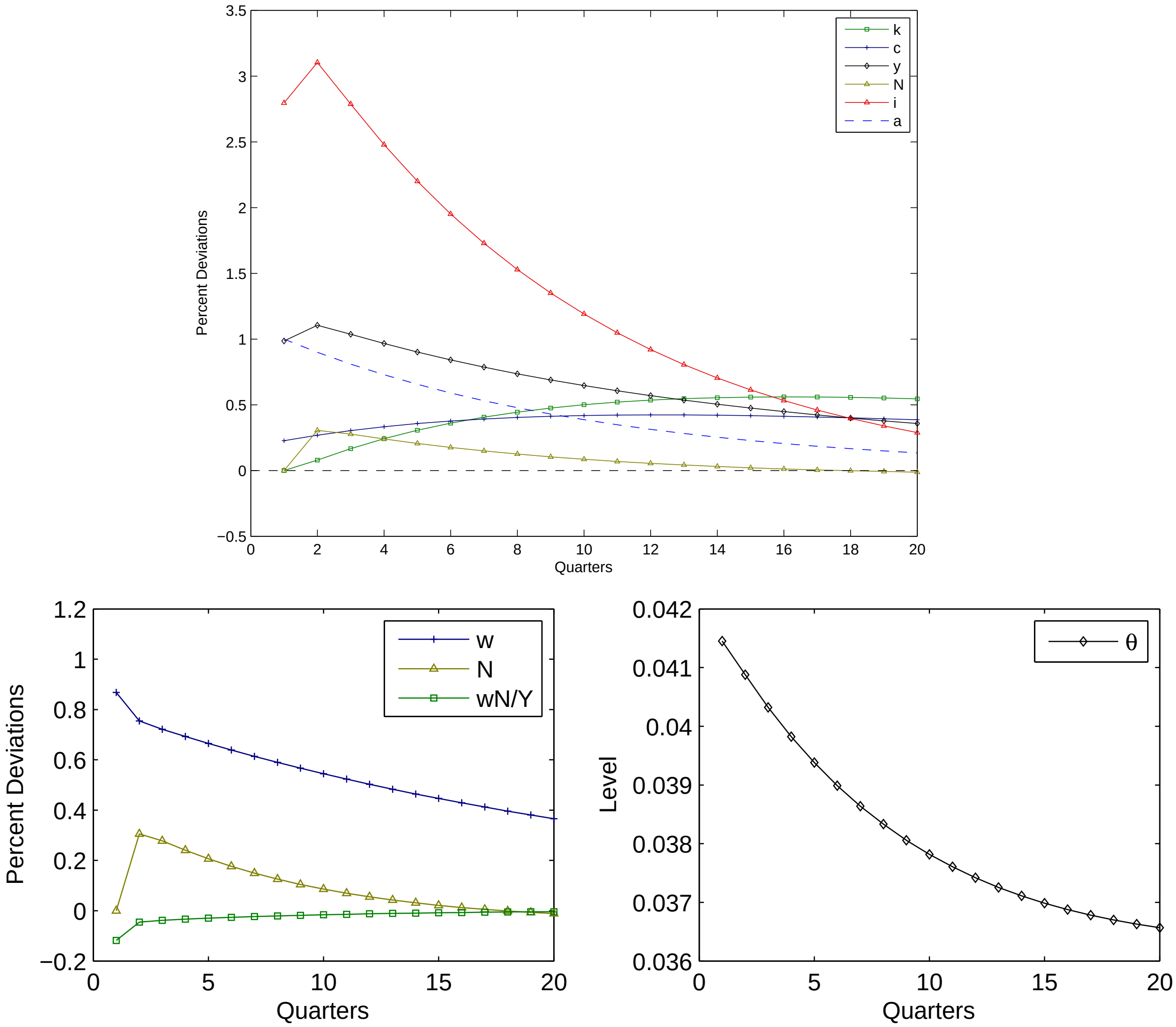
$$\hat{c}_t = E_t\hat{c}_{t+1}-(1-\beta/\gamma_x(1-\delta))\,E_t(\hat{y}_{t+1}-\hat{k}_{t+1}) \tag{64}$$

$$\hat{J}^n_t = \hat{V}^n_t \tag{65}$$

$$\bar{c}\hat{c}_t+\gamma_x\bar{k}\hat{k}_{t+1} = \hat{y}_t+(1-\delta)\bar{k}k_t \tag{66}$$

where $\widehat{mpl_t}$ is the log deviation of $\frac{\alpha y_t}{(1+\theta_t)N_t-\theta_t}$ from its steady state value. The model solved using the QZ decomposition in the matlab file **search.m**. Figure 1 shows the impulse

Figure 1: Impulse Response to a 1% positive shock to technology



response to a one percent positive innovation in productivity. Because a persistent increase in total factor productivity spurs investment into physical capital, consumption rises less than income. This moderates the response of the real wage, which now increases less than proportional to the increase in productivity. As a result, firms find it profitable to devote more resources to recruiting and expand employment. Hence, in contrast to the model of the previous section, technology shocks are no longer neutral for employment. Whereas the response of consumption and investment are similar to the standard RBC model, the labor market search model is able to generate cyclical fluctuations in (un)employment. However, the volatility of employment remains relatively small compared to US data. Table 1 shows some key business cycle moments based on band-pass filtered US data taken from Stock and Watson (1999) as well as their simulated model equivalents. In the data the relative stabndard deviation of employment is 0.88, whereas it is less than half as large in the model simulations. Moreover, the model still implies a highly procyclical real wage.

Table 1: **Business Cycle Moments**

variable x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\hat{y})}$	$\rho(x,\hat{y})$
	Stock and Watson (1999) BP-filtered, sample 1953-1996		
y	1.66	1.00	1.00
c	0.78	0.47	0.75
i	4.97	3.00	0.82
n	1.39	0.88	0.92
w	0.64	0.39	0.16
	Model, BP-filtered		
variable x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\hat{y})}$	$\rho(x,\hat{y})$
y	1.27	1.00	1.00
c	0.32	0.25	0.83
i	3.70	2.60	0.99
n	0.35	0.27	0.95
w	0.88	0.69	0.99

3 Generating Realistic Employment Volatility

This section discusses two adjustments to the labor market search model of the previous section that improve the ability of the model to generate realistic employment volatility in response to productivity shocks.

3.1 Wage Rigidity

Different from the centralized neoclassical labor market, labor search frictions generally imply a range of real wages at which both workers are willing to work and firms are willing to provide employment. The previous sections assumed that real wages arise from (Nash) bargaining between a matched worker and the firm in a bilateral monopoly situation. Any real wage within the bargaining set (no less than the worker's MRS but no more than the firm's MPL) is therefore a possible equilibrium wage. This indeterminacy was resolved by assuming that the real wage is a weighted average of the MRS and MPL, with the time invariant weight reflecting the relative bargaining power. This resulted in a strongly procyclical wage, dampening significantly the employment response to productivity shocks. Hall (2005) proposes to replace the surplus-splitting rule with the assumption of wage rigidity. At least at business cycle frequencies, this is perhaps not unrealistic given the approximate acyclicality of real wages in the data. This dramatically increases the employment response to shocks and leads to more realistic volatility of employment.

To verify this in the model of the previous section, replace the bargaining outcome in (48), or equivalently the wage equation in (57) with

$$w_t = X_t \bar{\bar{w}} \tag{67}$$

which simply states that real wages grow deterministically in proportion to a long run trend due to labor-enhancing technological progress. In the loglinear system, replace (65) with

$$\hat{w}_t = 0 \tag{68}$$

Figure 2 shows impulse responses to a one percent increase in productivity, whereas Table **??** shows the simulated moments. Both show that the assumption of perfectly rigid wages leads to significantly greater propagation as well as a quantitatively realistic relative volatility of employment.

Figure 2: Impulse Response to a 1% positive shock to technology

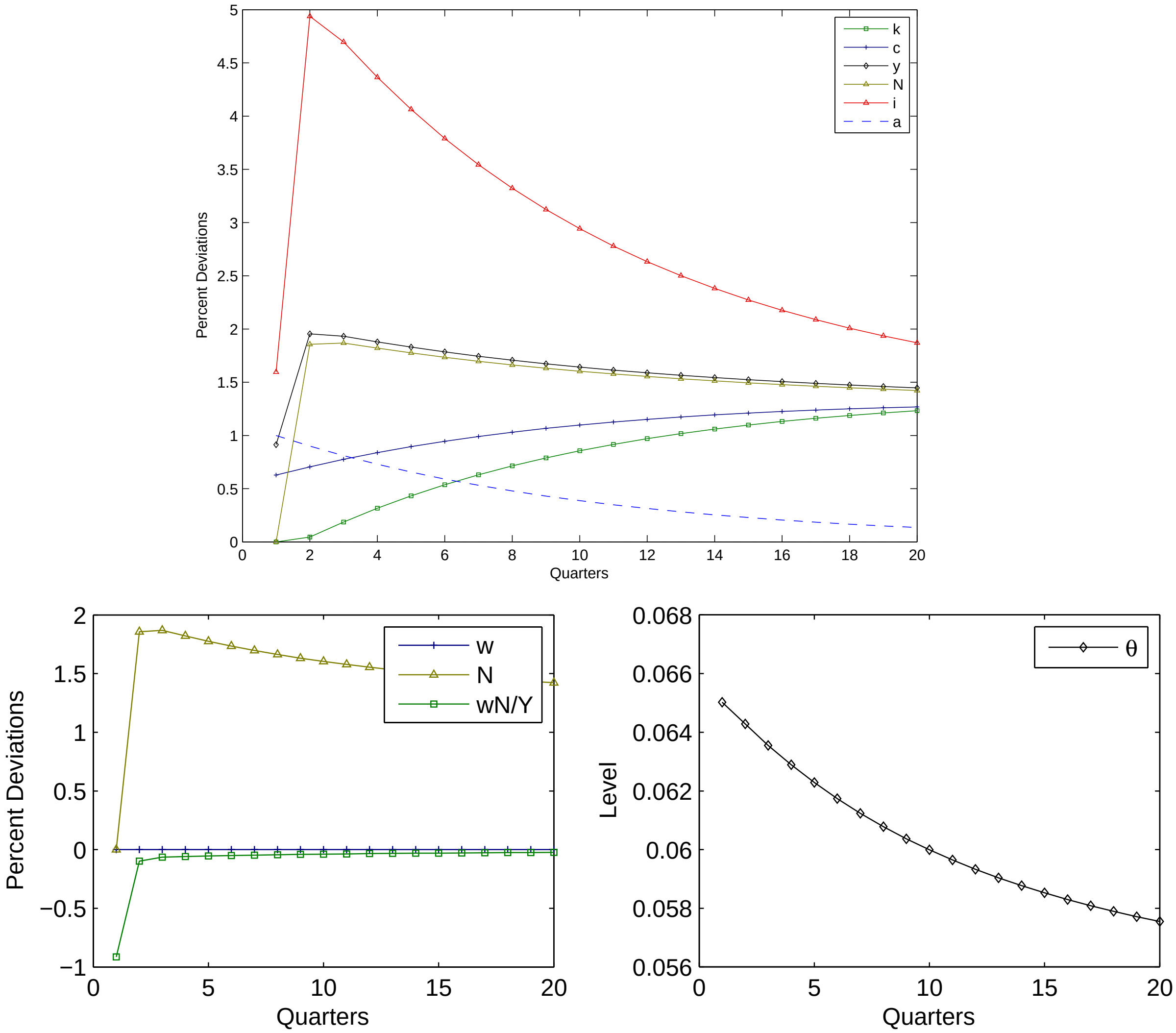


Table 2: **Business Cycle Moments: Rigid Wages**

	Model, BP-filtered		
variable x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\hat{y})}$	$\rho(x,\hat{y})$
y	2.18	1.00	1.00
c	0.78	0.36	0.95
i	5.63	2.57	0.99
n	2.14	0.98	0.97
w	0.00	0.00	0.00

3.2 Alternative Calibration

Hagedorn and Manovskii (2008) propose a different way of generating greater employment volatility that maintains the bargaining solution of the original model. The bargaining weight ζ in the original calibration was set rather arbitrarily to 0.5. Another way to dampen the increase of the real wage in response to a productivity shock is to assume that workers have little bargaining power. Hagedorn and Manovskii (2008) set $\zeta = 0.052$ and motivate this by the fact that wages are at best only moderately procyclical.

Figure 3 shows impulse responses to a one percent increase in productivity, whereas Table 3 shows the simulated moments in the model when $\zeta = 0.052$ (and recalibrating α, δ and γ to target the same moments as before). The lower bargaining weight leads to only small changes in wages but large adjustments in employment. The relative volatility of employment is close to the data. Note that the real wage remains strongly positively correlated with output, but the small elasticity of wages to productivity shocks implies that including other shocks in the model will easily result in acyclical wages.

Table 3: **Business Cycle Moments: Alternative Calibration**

	Model, BP-filtered		
variable x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\hat{y})}$	$\rho(x, \hat{y})$
y	1.78	1.00	1.00
c	0.43	0.24	0.81
i	5.23	2.94	0.99
n	1.13	0.64	0.97
w	0.50	0.28	0.90

Figure 3: Impulse Response to a 1% positive shock to technology