Money and Output:
Basic Facts and Flexible Price Models

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Contents

References

Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2003), 'Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem?', *Econometrica* **68**(5), 1151–1179.

Cooley, T. and Hansen, G. (1989), 'The inflation tax in a real business cycle model', *American Economic Review* **79**(4), 733–748.

Friedman, M. and Schwartz, A. J. (1963), *Monetary History of the United States, 1867-1960*, Princeton University Press.

King, R. G., Plosser, C. I. and Rebelo, S. T. (1988), 'Production, growth and business cycles: I. the basic neoclassical model', *Journal of Monetary Economics* **21**(2-3), 195–232.

King, R. and Plosser, C. (1984), 'Money, credit, and prices in a real business cycle', *American Economic Review* **74**(3), 363–380.

Kydland, F. E. and Prescott, E. C. (1990), 'Business cycles: Real facts and a monetary myth', *Quarterly Review, Federal Reserve Bank of Minneapolis* .

McCandless, G. T. and Weber, W. (1995), 'Some monetary facts', *Federal Reserve Bank of Minneapolis, Quarterly Review* .

Stock, J. H. and Watson, M. W. (1999), 'Business cycle fluctuations in us macroeconomic time series', *Handbook of Macroeconomics* **1**.

Figure 1: Definition of Monetary Aggregates

M1	**1392.3**
Currency	741.8
Travelers' checks	6.9
Demand deposits	327.0
Other checkable deposits	316.6
M2	**6775.2**
Components of M1	1392.3
Savings deposits, including MMDAs	3602.7
Small-denomination time deposits	1050.2
MMMFs (noninstitutional)	730.1

Note: Numbers may not add to totals shown owing to rounding.
Source: Federal Reserve Statistical Release H.6, June 22, 2006.
Data are not seasonally adjusted.

1 Monetary Facts

The previous chapter presented models with no role for money and no predictions for nominal variables. In this chapter we will analyze models that incorporate monetary factors that allow for the analysis of price-level determination, inflation and monetary policy and have implications for the behavior of nominal variables in the short and long run. The broad question that these models aim to address is whether money matters.[1] Cooley and Hansen (1989) give this fundamental question three different interpretations:

- Do money and the form of the money supply rule affect the nature and amplitude of the business cycle?
- How does anticipated inflation affect the long-run values of macroeconomic variables?
- What are the welfare costs associated with different money supply rules?

Early business cycle models that have addressed these questions have been heavily inspired by the monetarist tradition initiated by the empirical and theoretical work of Milton Friedman and Anna Schwartz, which attributed a great role to money for generating cyclical fluctuations. In this chapter we will address these questions in various flexible-price dynamic stochastic general equilibrium (DSGE) models.

[1]For the definitions of the most common monetary aggregates, see Figure 5.

Here are some monetary facts, which one should stress do **not** suggest any direction of causation whatsoever:

1.1 Long run monetary facts

- Growth rates of monetary aggregates and inflation are extremely highly and positively correlated across countries and within countries in the long run, regardless of the definition of money. In the long run, there is a one-to-one relationship between money growth and inflation.

- Long-run average growth rates of monetary aggregates and real output are not correlated across most countries. However, this fact is not entirely robust across subsamples of countries. McCandless and Weber (1995) find a positive relation for OECD countries and a negative relation for Latin American countries.

- McCandless and Weber (1995) find that inflation rates are not correlated with real output growth across countries. Other studies present some evidence for a slight negative correlation.

Figure 2: Correlation of money growth and inflation, source: McCandless and Weber (1995)

Table 1

Correlation Coefficients for Money Growth and Inflation*

Based on Data From 1960 to 1990

Sample	Coefficient for Each Definition of Money		
	M0	M1	M2
All 110 Countries	.925	.958	.950
Subsamples			
21 OECD Countries	.894	.940	.958
14 Latin American Countries	.973	.992	.993

*Inflation is defined as changes in a measure of consumer prices.
Source of basic data: International Monetary Fund

Table 2

Previous Studies of the Relationship Between Money Growth and Inflation

Author (and Year Published)	Study Characteristics: Time Series: Money	Study Characteristics: Time Series: Inflation	Study Characteristics: Countries	Study Characteristics: Time Period	Study Characteristics: Data Frequency	Finding
Vogel (1974)	Currency + Demand deposits	Consumer prices	16 Latin American countries	1950–69	Annual	Proportionate changes in inflation rate within two years of changes in money growth
Lucas (1980)	M1	Consumer prices	United States	1955–75	Annual	Strong positive correlation: Coefficient closer to one the more filter stresses low frequencies
Dwyer and Hafer (1988)	n.a.	GDP deflator	62 countries	1979–84	Five-year averages	Strong positive correlation
Barro (1990)	Hand-to-hand currency	Consumer prices	83 countries	1950–87	Full-period averages	Strong positive association
Pakko (1994)	Currency + Bank deposits	Consumer prices	13 former Soviet republics	1992 and 1993	Four-quarter averages	Positive relationship
Poole (1994)	Broad money	n.a.	All countries in World Bank tables	1970–80 and 1980–91	Annual averages	Strong positive correlation
Rolnick and Weber (1994)	Various	Various	9 countries	Various	Long-period averages	Strong positive correlation for fiat money regimes

n.a. = not available

Figure 3: Correlation of money growth and real output growth, source: McCandless and Weber (1995)

Table 3

Correlation Coefficients for Money Growth and Real Output Growth*

Based on Data From 1960 to 1990

	Coefficient for Each Definition of Money		
Sample	M0	M1	M2
All 110 Countries	–.027	–.050	–.014
Subsamples			
21 OECD Countries	.707	.511	.518
14 Latin American Countries	–.171	–.239	–.243

*Real output growth is calculated by subtracting changes in a measure of consumer prices from changes in nominal gross domestic product.

Source of basic data: International Monetary Fund

Table 4

Previous Studies of the Relationship Between Money Growth and Real Output Growth

	Study Characteristics					
	Time Series					
Author (and Year Published)	Money	Output	Countries	Time Period	Data Frequency	Finding
Kormendi and Meguire (1985)	M1	Real GDP	47 countries	1950–77	Period averages	Negative correlation
Geweke (1986)	M2, M1	NNP, industrial production	United States	1870–1978, Postwar period	Annual, monthly	Money superneutral
Dwyer and Hafer (1988)	n.a.	Real GDP and GNP	62 countries	1979–84	Five-year averages	Slight negative correlation (not statistically significant)
Poirier (1991)	M1	Real GDP	47 countries	1873	Annual	Money neutral in some countries, not in others

n.a. = not available

Figure 4: Correlation of inflation and real output growth, source: McCandless and Weber (1995)

Table 5

Correlation Coefficients for Inflation and Real Output Growth*

Based on Data From 1960 to 1990

	Coefficient With Outlier**	
Sample	Included	Excluded
All 110 Countries	–.243	–.101
Subsamples		
21 OECD Countries	.390	.390
14 Latin American Countries	—	–.342

*Inflation is defined as changes in a measure of consumer prices. Real output growth is calculated by subtracting those inflation rates from changes in nominal gross domestic product.

**The outlier is Nicaragua.

Source of basic data: International Monetary Fund

Table 6

Previous Studies of the Relationship Between Inflation and Real Output Growth

	Study Characteristics					
	Time Series					
Author (and Year Published)	Inflation	Output	Number of Countries	Time Period	Data Frequency	Finding
Fischer (1983)	n.a.	n.a.	53	1961–73, 1973–81	Annual	Negative contemporaneous relationship; positive correlation with one lag
Kormendi and Meguire (1985)	Consumer prices	Real GDP	47	1950–77	Period averages	Negative correlation
Fischer (1991)	GDP deflator	GDP	73	1970–85	Annual	Negative relationship
Altig and Bryan (1993)	GDP deflator	Per capita GDP	54 and 73	1960–88	Annual	Negative correlation
Ericsson, Irons, and Tryon (1993)	GDP deflator	GDP	102	1960–89	Annual	Weak negative correlation
Barro (1995)	Consumer prices	Per capita real GDP	78, 89, and 84	1965–90	Five- or ten-year averages	Negative correlation

n.a. = not available

1.2 Short run monetary facts

- M0 (= the monetary base, i.e. currency in circulation + total reserves held by banks), M1 and M2 are all pretty volatile and procyclical.

- M0, M1 and M2 velocities are all volatile and procyclical. Note the velocity of money V is

$$V = PY/M$$

  where PY is nominal GDP and P is the price level.

- M0, M1 and M2 lead real output (see Friedman and Schwartz (1963)). Stock and Watson (1999) find the log level of nominal M2 is procyclical with a lead of two quarters, and the nominal monetary base is weakly procyclical and also leading. In contrast, the growth rates of nominal M2 and the nominal monetary base are countercyclical and lagging. Kydland and Prescott (1990) disagree that monetary aggregates are leading indicators (monetary myth).

- Stock and Watson (1999) find that the cyclical component of the price level, measured for instance by the CPI, is countercyclical and leads the cycle by approximately two quarters. The cyclical components of inflation rates instead are strongly procyclical and lag the business cycle. The nominal wage index exhibits a pattern quite similar to the CPI price level. Real wages have essentially no contemporaneous comovement with the business cycle.

- Growth rates of monetary aggregates and inflation are not correlated in the short run, except in episodes of hyperinflation, during which the relationship is one-to-one.

- Short term nominal interest rates are procyclical.

Figure 5: source: Kydland and Prescott (1990)

Table 4

Cyclical Behavior of U.S. Monetary Aggregates and the Price Level

Deviations From Trend of Money Stock, Velocity, and Price Level
Quarterly, 1954–1989

		Cross Correlation of Real GNP With										
Variable *x*	Volatility (% Std. Dev.)	*x*(*t*–5)	*x*(*t*–4)	*x*(*t*–3)	*x*(*t*–2)	*x*(*t*–1)	*x*(*t*)	*x*(*t*+1)	*x*(*t*+2)	*x*(*t*+3)	*x*(*t*+4)	*x*(*t*+5)
Nominal Money Stock*												
Monetary Base	0.88	–0.12	0.02	0.14	0.25	0.36	0.41	0.40	0.37	0.32	0.28	0.26
M1	1.68	0.01	0.12	0.23	0.33	0.35	0.31	0.22	0.15	0.09	0.07	0.07
M2	1.51	0.48	0.60	0.67	0.68	0.61	0.46	0.26	0.05	–0.15	–0.33	–0.46
M2 – M1	1.91	0.53	0.63	0.67	0.65	0.56	0.40	0.20	–0.01	–0.21	–0.39	–0.53
Velocity*												
Monetary Base	1.33	–0.26	–0.15	0.00	0.22	0.40	0.59	0.50	0.37	0.22	0.08	–0.08
M1	2.02	–0.24	–0.19	–0.12	–0.01	0.14	0.31	0.32	0.27	0.20	0.10	0.00
M2	1.84	–0.63	–0.59	–0.48	–0.29	–0.05	0.24	0.34	0.40	0.43	0.44	0.43
Price Level												
Implicit GNP Deflator	0.89	–0.50	–0.61	–0.68	–0.69	–0.64	0.55	–0.43	–0.31	–0.17	–0.04	0.09
Consumer Price Index	1.41	–0.52	–0.63	–0.70	–0.72	–0.68	0.57	–0.41	–0.24	–0.05	0.14	0.30

*Based on quarterly data, 1959:1–1989:4.
Source of basic data: Citicorp's Citibase data bank

2 Monetary Models

The fundamental problem in monetary models is how to model the demand for fiat money. Seeing fiat money as an asset, it may assume the role as a *store of value*. However, money is almost always dominated in return by other assets. A second potential motive for holding money is as a *unit of account*, but any other good can serve that purpose. A third, more plausible reason for holding money is that it facilitates economic transactions as a *medium of exchange*, for instance by avoiding the *double coincidence of wants*. Although there is a sizeable literature in which money fulfills one of the above roles in very well-specified way, many macroeconomic business cycle models take a more ad-hoc approach. This chapter covers three widely used models: the Money-in-Utility (MIU) model, the Cash-in-Advance (CIA) Model and the Shopping Time (ST) Model

2.1 A Money-in-the-Utility (MIU) Model

2.1.1 A Basic MIU model

Households The economy is populated by a representative household with preferences represented by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, m_t) \ , \ 0 < \beta < 1 \tag{1}$$

where $C_t > 0$ is commodity consumption in period t, $m_t = \frac{M_t}{P_t}$ is the real value of money holdings, $M_t > 0$ denotes nominal money balances and P_t is the nominal price level. Assume that $u_c > 0$ and $u_m > 0$ and that $u(C, m)$ is strictly concave in both arguments, twice continuously differentiable and satisfies $\lim_{m \to 0} u_m(C, m) = \infty$. Note that (1) implies that, holding constant the path of real consumption C_t for all t, the individual's utility is increased by an increase in real money holdings. Even though the money holdings are never used to purchase consumption, they yield utility. The household supplies its unitary endowment of time inelastically in the labor market.

The final good can be either consumed or used for investment, i.e. it can be added to the capital stock K_t, which evolves according to

$$K_{t+1} = I_t + (1 - \delta) K_t \ , \ 0 < \delta < 1 \tag{2}$$

where I_t denotes gross investment.

The household's period budget constraint is:

$$C_t + I_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} \leq w_t + r_t K_t + (1+R_{t-1})\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t}$$

or

$$C_t + I_t + b_t + m_t \leq w_t + r_t K_t + \frac{1+R_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + t_t$$

where $b_t = \frac{B_t}{P_t} > -\bar{b}$ denote real holdings of a one period uncontingent government issued bond, R_t is the nominal interest rate on the bond, $t_t = \frac{T_t}{P_t}$ denote any real lump-sum transfers by the government and π_t is the rate of inflation.[2] Without loss of generality, we abstract from modeling the market for firms' shares (see previous chapter). The assets available to the households are physical capital, government bonds and money balances. The household's problem is to choose the real quantities $\{C_t, b_t, m_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (1) subject to the law of motion for capital, the budget constraints and taking as given inflation and nominal interest rates $\{\pi_t, R_t\}_{t=0}^{\infty}$ as well as the real factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, the real transfers $\{t_t\}_{t=0}^{\infty}$ and the initial capital stock K_0 and initial real bond and money holdings and nominal interest rate b_{-1}, m_{-1}, R_{-1}.[3]

Firms There is only one final good in the economy that is produced by firms according to a production technology given by

$$Y_t = A_t (K_t^d)^{1-\alpha} (N_t^d)^{\alpha}$$

where K_t^d is the capital input and N_t^d is labor input, which are rented in competitive markets at real prices w_t and r_t respectively. Total factor productivity evolves according to the following stochastic process,

$$\begin{aligned} A_t &= \bar{A} e^{a_t} \\ a_t &= \rho^a a_{t-1} + \epsilon_t^a \end{aligned} \tag{3}$$

where ϵ_t^a is a white noise random variable with standard deviation σ_ϵ^a and $0 < \rho^a < 1$ measures the shock persistence. The firm's problem in each period t is to choose N_t and

[2] Note the constraint $b_t > -\bar{b}$ on bond holdings, which is a sufficient condition to exclude *Ponzi-scheme* solutions to the households' problem. For discussion, see Wouter Denhaan's lecture notes.

[3] We abstract from the labor leisure choice since it is assumed that the household supplies one unit of labor inelastically.

K_t to maximize real profits

$$Y_t - r_t K_t^d - w_t N_t^d$$

taking the factor prices as given.

Government The government is the monopoly supplier of money, which it uses to finance the lump-sum transfers and the net debt obligations to the household. The government budget constraint is therefore

$$\frac{M_t^s}{P_t} - \frac{M_{t-1}^s}{P_t} + \frac{B_t^s}{P_t} = \frac{T_t}{P_t} + (1 + R_{t-1})\frac{B_{t-1}^s}{P_t}$$

or

$$m_t^s - \frac{m_{t-1}^s}{1+\pi_t} + b_t^s - \frac{1 + R_{t-1}}{1+\pi_t} b_{t-1}^s = t_t$$

According to this government budget constraint, the transfers T_t to the households and the interest payments $R_{t-1}B_{t-1}^s$ on outstanding government debt must be funded either by additional borrowing $\frac{B_t^s - B_{t-1}^s}{P_t}$ or by expanding the real money supply $\frac{M_t^s - M_{t-1}^s}{P_t}$ ("printing money", "seigniorage"). For now we will assume for simplicity that $B_{-1} = 0$ and $B_t^s = 0$ for all t, such that the government budget constraint reduces to

$$t_t = m_t^s - \frac{m_{t-1}^s}{1+\pi_t}$$

Assume that the growth rate of the money supply in deviation of the steady state growth rate, denoted by $\theta_t = \frac{M_t}{M_{t-1}} - \mu - 1$, is exogenous and evolves according to

$$\theta_t = \rho^\theta \theta_{t-1} + \epsilon_t^\theta$$

where $\mu > 0$ is the average growth rate of the money supply, ϵ_t^θ is a white noise random variable with standard deviation σ_ϵ^θ and $0 < \rho^\theta < 1$ measures the shock persistence.

Equilibrium An equilibrium is defined as an infinite sequence of allocations of consumption, capital, labor inputs and real bond and money holdings and a system of price sequences containing the real factor prices, a nominal interest rate and inflation such that for all t:

- The goods market clears: $C_t + I_t = Y_t$
- The market for money clears: $m_t^s = m_t$
- The bond market clears: $b_t^s = b_t = 0$

- The factor markets clear. $K_t^d = K_t$ and $N^d = 1$.

- The government satisfies its budget constraint.

and the households and firms solve their respective problems for every sequence of innovations to productivity and the money growth rate.

Money Demand In equilibrium, the following conditions must be satisfied in every period t at an interior solution:

$$-u_c(C_t, m_t) + \beta E_t \left[u_c(C_{t+1}, m_{t+1}) \left((1-\alpha)A_{t+1}K_{t+1}^{-\alpha} + 1 - \delta\right)\right] = 0 \tag{4a}$$

$$-u_c(C_t, m_t) + \beta E_t \left[u_c(C_{t+1}, m_{t+1}) \frac{1+R_t}{1+\pi_{t+1}}\right] = 0 \tag{4b}$$

$$u_m(C_t, m_t) + \beta E_t \left[u_c(C_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}}\right] - u_c(C_t, m_t) = 0 \tag{4c}$$

together with $C_t = A_t K_t^{1-\alpha} + (1-\delta)K_t - K_{t+1}$ and the government budget constraint. Equation (4a) is the familiar Euler equation describing the optimal consumption-investment choice, (4b) is the bond Euler equation, (4c) is the money demand equation.
The transversality conditions are

$$\begin{aligned}
\lim_{T\to\infty} \beta^T E_t \left[u_c(C_T, m_T) K_{T+1}\right] &= 0 \\
\lim_{T\to\infty} \beta^T E_t \left[u_c(C_T, m_T) b_T\right] &= 0 \\
\lim_{T\to\infty} \beta^T E_t \left[u_c(C_T, m_T) m_T\right] &= 0
\end{aligned}$$

Let's have a closer look at the money demand equation:

$$\begin{aligned}
u_m(C_t, m_t) &= -\beta E_t \left[u_c(C_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}}\right] + u_c(C_t, m_t) \\
\frac{u_m(C_t, m_t)}{u_c(C_t, m_t)} &= 1 - \beta E_t \left[\frac{u_c(C_{t+1}, m_{t+1})}{u_c(C_t, m_t)} \frac{1}{1+\pi_{t+1}}\right] \\
\frac{u_m(C_t, m_t)}{u_c(C_t, m_t)} &= 1 - \frac{1}{1+R_t} && \text{by (4b)} \\
\frac{u_m(C_t, m_t)}{u_c(C_t, m_t)} &= \frac{R_t}{1+R_t} && (5)
\end{aligned}$$

The term $\frac{R_t}{1+R_t}$ constitutes the "price of money" in the sense that it is the dollar opportunity cost of holding an additional unit of money. Instead the household could purchase a bond and earn interest tomorrow, the real present value of which today is $\frac{R_t}{1+R_t}$. Equation (5) implicitly defines the money demand and taken together with the exogenous money supply process may remind you of the "LM curve" describing money market equilibrium in undergraduate macro. Similarly, you can think of (4a), (4b) together with the resource constraint as determining a dynamic version of the "IS curve" describing goods market equilibrium.

The Deterministic Steady State Consider for a moment a different version of the model in which there are no stochastic shocks. In a *deterministic* steady state

$$\beta\left((1-\alpha)\bar{A}\bar{K}^{-\alpha}+1-\delta\right) \quad = \quad 1 \tag{6}$$

The steady state level of capital, consumption, investment and output in the nonstochastic model are determined by equation (6), the law of motion for capital $\bar{I}=\delta K$, $\bar{Y}=\bar{A}\bar{K}^{1-\alpha}$ and the resource constraint $\bar{C}+\bar{I}=\bar{Y}$. They are independent of any utility parameter other than β and do not depend on the rate of inflation or the money growth rate.
Next, note that to ensure that a steady state monetary equilibrium exists in which $\bar{m}>0$ is constant, there must exist a positive value of $\bar{m}$ that solves

$$u_m(\bar{C},\bar{m})=\left(1-\frac{\beta}{1+\mu}\right)u_c(\bar{C},\bar{m})$$

Depending on the instantaneous utility function, there might be no solution, a unique solution or even multiple solutions. Consider for instance the simpler case where utility is additively separable, such that $u(C,m)=u^1(C)+u^2(m)$ and

$$u_m^2(\bar{m}) \quad = \quad \left(1-\frac{\beta}{1+\mu}\right)u_c^1(\bar{C})>0$$

A unique solution where $\bar{m}>0$ exists given that our earlier assumptions imply that $\lim_{m\to 0}u_m^2(m)=\infty$, $u_{mm}^2<0$ and provided there exists some $\tilde{m}$ such that $u_m^2(\tilde{m})<\left(1-\frac{\beta}{1+\mu}\right)u_c^1(\bar{C})$. In order to analyze the dynamics of the nominal price level around the deterministic steady state, consider that

$$
\begin{aligned}
\frac{\beta}{1+\pi_{t+1}} u_c^1(\bar{C}) &= u_c^1(\bar{C}) - u_m^2(m_t) \\
\frac{\beta}{1+\pi_{t+1}} u_c^1(\bar{C}) M_{t+1} &= \left(u_c^1(\bar{C}) - u_m^2(m_t)\right) M_{t+1} \\
\frac{\beta}{1+\pi_{t+1}} u_c^1(\bar{C}) \frac{M_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} &= \left(u_c^1(\bar{C}) - u_m^2(m_t)\right) (1+\mu) \frac{M_t}{P_t} \\
\frac{\beta}{1+\mu} u_c^1(\bar{C}) m_{t+1} &= \left(u_c^1(\bar{C}) - u_m^2(m_t)\right) m_t \\
m_{t+1} &= \frac{1+\mu}{\beta} \left(1 - \frac{u_m^2(m_t)}{u_c^1(\bar{C})}\right) m_t \equiv \Phi(m_t) \qquad (7)
\end{aligned}
$$

Price level determinacy for a given exogenous path of the money supply M_t depends on the properties of the function Φ. First notice that we can rule out any price paths solving equation (7) that lead to m_s going to infinity for $s \to \infty$ because of the transversality condition. This rules out solutions where the price level does not grow at least at the same rate as the money supply, or in other words, inflation is at least the money growth rate. Suppose $\lim_{m \to 0} \Phi(m) < 0$ (or equivalently $\lim_{m \to 0} u_m^2 m > 0$), then given all our earlier assumptions, there can only exist one solution, since real money balances cannot be negative. In this case, the price level P_t is determined, and as a jump variable, always adjusts such that $m = \bar{m} > 0$. However, if $\lim_{m \to 0} u_m^2 m = 0$ such that $\lim_{m \to 0} \Phi(m) = 0$, there exist price paths for which money balances are positive for all s and where $m_s \to 0$ as $s \to 0$. These solutions, where inflation exceeds the rate of money growth, are characterized by *speculative hyperinflations*. They differ from regular hyperinflations because they are not caused by high growth rates of the money supply. The steady state price level and inflation rates are not determined in those cases. Unfortunately, the condition $\lim_{m \to 0} u_m^2 m > 0$ necessary to rule out these hyperinflationary solutions implies that $\lim_{m \to 0} u^2(m) = -\infty$. Money must be such an important good that utility goes to minus infinity when real balances drop to zero! Nevertheless we will assume that this condition is satisfied, unless otherwise mentioned.

Consider the following definitions:

- **Neutrality**: A model has the property of neutrality when a once and for all change in the *level* of the money supply changes the price level proportionally such that real money holdings are constant.

- **Superneutrality**: A model has the property of superneutrality when a change in the *growth rate* of the money supply only affects real money balances but leaves all other real variables unchanged.

The MIU model without stochastic shocks displays *long run* superneutrality because the money growth rate μ does not affect the steady state level of capital, output and consumption. Note however that except for very specific utility functions, there is non-superneutrality during the *transition to* the steady state. The MIU model without stochastic shocks also displays monetary neutrality both in the long and short run. Monetary neutrality is a general property of flexible price monetary models. In a later chapter, we will see models with nominal rigidities (sticky price models) in which money is not neutral.

Dynamics in the stochastic model Consider again the stochastic model, in which there are random innovations to productivity and monetary growth. We will consider the following parametrization of the utility function

$$u(C_t, m_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \phi \frac{m_t^{1-\chi} - 1}{1-\chi}$$

where $\sigma > 0$, $\chi > 0$, $\phi > 0$. The first order necessary conditions are

$$-C_t^{-\sigma} + \beta E_t \left[C_{t+1}^{-\sigma} \left((1-\alpha) A_{t+1} K_{t+1}^{-\alpha} + 1 - \delta \right) \right] = 0 \tag{8a}$$

$$-C_t^{-\sigma} + \beta E_t \left[C_{t+1}^{-\sigma} \frac{1+R_t}{1+\pi_{t+1}} \right] = 0 \tag{8b}$$

$$\phi m_t^{-\chi} - \frac{R_t}{1+R_t} C_t^{-\sigma} = 0 \tag{8c}$$

$$A_t K_t^{1-\alpha} + (1-\delta) K_t - K_{t+1} = C_t \tag{8d}$$

where

$$\frac{M_t}{M_{t-1}} = 1 + \mu + \theta_t \tag{8e}$$

$$\theta_t = \rho^\theta \theta_{t-1} + \epsilon_t^\theta \tag{8f}$$

$$\log(A_t) = (1-\rho^a) \log(\bar{A}) + \rho^a \log A_{t-1} + \epsilon_t^a \tag{8g}$$

The loglinearized dynamics around the deterministic steady state are described by

$$-\sigma\hat{c}_t = -\sigma E_t\hat{c}_{t+1} + (1-\beta(1-\delta))\, E_t(a_{t+1} - \alpha\hat{k}_{t+1}) \tag{9a}$$

$$-\sigma\hat{c}_t = -\sigma E_t\hat{c}_{t+1} + \hat{R}_t - E_t\hat{\pi}_{t+1} \tag{9b}$$

$$-\chi\hat{m}_t = -\sigma\hat{c}_t + \left(\frac{\beta}{1+\mu-\beta}\right)\hat{R}_t \tag{9c}$$

$$s_c\hat{c}_t + \frac{s_i}{\delta}\hat{k}_{t+1} = a_t + \left((1-\alpha) + s_i\frac{1-\delta}{\delta}\right)\hat{k}_t \tag{9d}$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + \theta_t \tag{9e}$$

$$\theta_t = \rho^\theta\theta_{t-1} + \epsilon_t^\theta \tag{9f}$$

$$a_t = \rho^a a_{t-1} + \epsilon_t^a \tag{9g}$$

Watch out: for the nominal interest and inflation rate, we deviate from our standard definition of the hat variables and define $\hat{R}_t = (R_t - \bar{R})/(1+\bar{R})$ and $\hat{\pi}_t = (\pi_t - \bar{\pi})/(1+\bar{\pi})$.

Calibration The following parameters appear in the equations characterizing the model dynamics in the neighborhood of the deterministic steady state: α, β, δ, σ, $\bar{A}$, ρ_a, σ^a, ϕ, χ, μ, ρ^θ, σ^θ. Some of these parameters are familiar from the RBC model of the last chapter, and hence we will adopt the same values as before: $\alpha = 0.58$, $\beta = 0.988$, $\delta = 0.025$, $\sigma = 1$ $\rho_a = 0.9$ and $\sigma_a = 0.01$. $\bar{A}$ is chosen such that the steady state level of output is unity. However, we need to choose values for the new parameters ϕ, χ, μ, ρ^θ and σ^θ that characterize money demand and the exogenous process for the money growth rate. Using M1 as their money measure, Cooley and Hansen (1989) estimate the following process for money growth using data with quarterly frequency:

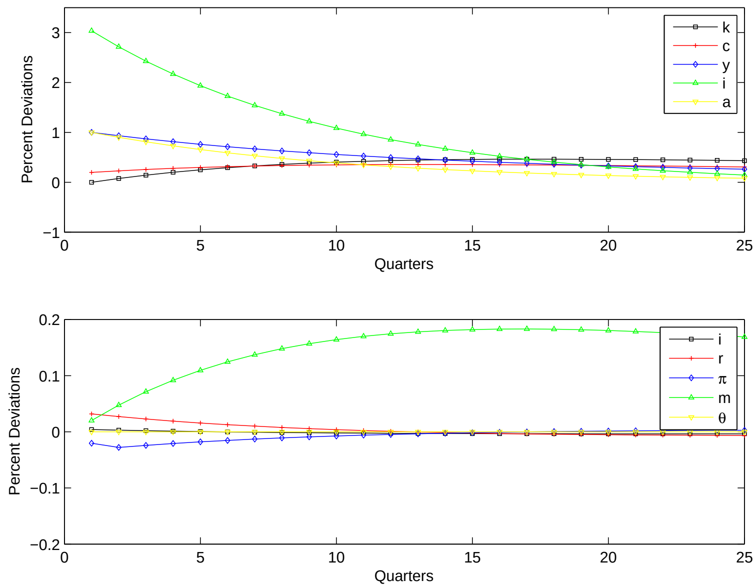
$$\Delta\log(M_t) = 0.00798 + 0.481\Delta\log(M_{t-1})$$

and $\sigma^\theta = 0.009$, which implies that $\rho^\theta = 0.481$ and $1+\mu = 1 + \frac{0.00798}{1-\rho^\theta} = 1.015$, i.e. a quarterly rate of money growth of 1.5%. An important parameter for the model dynamics is χ. Note that we can write the loglinearized version of money demand as

$$\hat{m}_t = \frac{\sigma}{\chi}\hat{c}_t - \frac{1}{\chi}\left(\frac{\beta}{1+\mu-\beta}\right)\hat{R}_t$$

The *interest rate elasticity of money demand* is therefore $-\frac{1}{\chi}\left(\frac{\beta}{1+\mu-\beta}\right)$. The empirical money demand literature however usually focuses on the interest rate *semi*-elasticity of

Figure 6: Response to a +1% technology shock



money demand

$$\frac{\partial \log(m_t)}{\partial(1+R_t)} = -\frac{1}{4\chi}\left(\frac{\beta}{1+\mu-\beta}\right)\frac{\beta}{1+\mu}$$

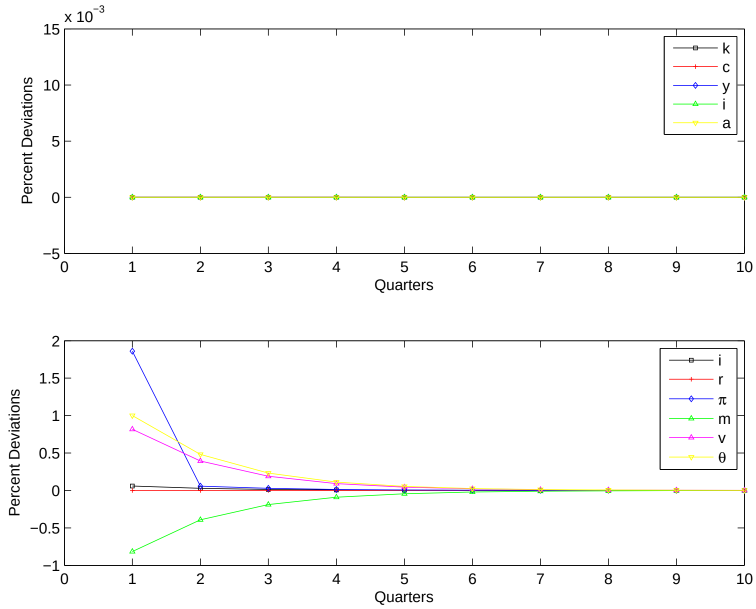
This expression takes into account that the time period of the model is a quarter, whereas the elasticity is measured with respect to the annualized interest rate. Many studies estimate different absolute values for the interest rate semi-elasticity of money demand, usually ranging from < 1 ($\chi > 10$) up to 8 ($\chi \approx 1$). We will take an intermediate value of $\chi = 1/0.39$, which is the estimate of Chari, Kehoe and McGrattan (2003). Finally, ϕ is chosen in order to match velocity $\bar{v} = \frac{\bar{y}}{\bar{m}}$ (i.e. the average ratio of nominal GDP to M1) of about $1/0.16$ from the steady state money demand relationship

$$\phi = \left(\frac{\bar{m}}{\bar{y}}\right)^{\chi} \bar{C}^{-\sigma}\frac{1+\mu-\beta}{1+\mu}\bar{y}^{\chi}$$

The model is solved in **MIUmodel.m** using the QZ decomposition. Figure 6 displays the impulse responses of the key macroeconomic variables to a 1% positive innovation in technology, and Figure 7 shows the impulse responses to a 1% positive innovation in money growth.

The first noteable feature of the model is the *classical dichotomy*. The real allocations are independent of monetary shocks (short-run superneutrality). Notice that equations

Figure 7: Response to a +1% money growth shock



(9a), (9d) and (9f) describe the dynamics of consumption and capital independently of the other equations. After a technology shock, the response of inflation, real money holdings, nominal interest rates and money velocity are consistent with the data. Besides the lack of real effects after a monetary expansion, there are two other dimensions in which the model does poorly. First, there is no dominating "liquidity effect" after a monetary expansion, which means that because of higher expected inflation, an increase in money supply growth leads to higher instead of lower nominal interest rates. Second, inflation is not persistent given our (realistic) choice for ρ^{θ}. These two shortcomings together with the lack of real effects are inconsistent with most of the empirical literature on monetary shocks that will be discussed in the next chapter. A final unattractive feature is that, when we introduce technological progress, the model is inconsistent with balanced growth unless $\chi = \sigma$. To see why, note that in the money demand equation (8c), growth in consumption cannot be reconciled with growth in real money balances in a way that leads to a stationary velocity of money.

2.1.2 An Extended MIU model with Nonseparable Preferences and Elastic Labor Supply

This section addresses some of the weaknesses of the basic MIU model by introducing an elastic labor supply and by assuming non-separable preferences of the form

$$u(C_t, m_t, 1-N_t) = \frac{\left(\left(C_t^{1-\chi} + \phi m_t^{1-\chi}\right)^{\frac{1}{1-\chi}} (1-N_t)^{\eta}\right)^{1-\sigma}}{1-\sigma}$$

Because consumption and real money holdings are no longer separable, changes in m_t will in general affect the marginal utility of consumption and leisure. These preferences are also appealing because they are consistent with balanced growth. The first order necessary conditions are now

$$\begin{aligned}
-\lambda_t + \left(C_t^{1-\chi} + \phi m_t^{1-\chi}\right)^{\frac{\chi-\sigma}{1-\chi}} (1-N_t)^{(1-\sigma)\eta} C_t^{-\chi} &= 0 \\
-\lambda_t + \beta E_t\left[\lambda_{t+1}\left((1-\alpha)A_{t+1}\left(\frac{K_{t+1}}{N_{t+1}}\right)^{-\alpha} + 1 - \delta\right)\right] &= 0 \\
-\left(C_t^{1-\chi} + \phi m_t^{1-\chi}\right)^{\frac{1-\sigma}{1-\chi}} \eta (1-N_t)^{(1-\sigma)\eta - 1} + \lambda_t \alpha A_t \left(\frac{K_t}{N_t}\right)^{1-\alpha} &= 0 \\
-\lambda_t + \beta E_t\left[\lambda_{t+1}\frac{1+R_t}{1+\pi_{t+1}}\right] &= 0 \\
\phi\left(\frac{m_t}{C_t}\right)^{-\chi} - \frac{R_t}{1+R_t} &= 0 \\
A_t K_t^{1-\alpha} N_t^{\alpha} + (1-\delta)K_t - K_{t+1} &= C_t
\end{aligned}$$

Deterministic Steady State Note that now the deterministic steady state levels of capital, output, etc. are no longer independent of the money growth rate. The reason is that labor supply is now affected by real money balances. The optimality condition for the labor-leisure can be written as

$$\eta \frac{C_t^{1-\chi} + \phi m_t^{1-\chi}}{(1-N_t)C_t^{-\chi}} = w_t$$

where w_t is the real wage. Unless $\chi = 1$, the steady state level of real balances, which in turn depends on the average money growth rate μ through the money demand equation, changes the steady-state labor supply. Higher μ lowers the real demand for money, which as long as $\chi > 1$ raises the marginal utility of leisure relative to the marginal utility of consumption. Therefore higher μ and higher steady state inflation lowers labor input and output in the deterministic steady state. Money is no longer superneutral.

Dynamics in the stochastic model The loglinearized dynamics around the deterministic steady state are now described by

$$\begin{aligned}
\hat{\lambda}_t &= -(1-\sigma)\eta\frac{\bar{N}}{1-\bar{N}}\hat{n}_t + \left(-\chi + \frac{(\chi-\sigma)\bar{C}^{1-\chi}}{\bar{C}^{1-\chi}+\phi\bar{m}^{1-\chi}}\right)\hat{c}_t + \left(\frac{(\chi-\sigma)\phi\bar{m}^{1-\chi}}{\bar{C}^{1-\chi}+\phi\bar{m}^{1-\chi}}\right)\hat{m}_t \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + (1-\beta(1-\delta))\,E_t(a_{t+1} - \alpha\hat{k}_{t+1} + \alpha\hat{n}_{t+1}) \\
a_t - (1-\alpha)\hat{n}_t + (1-\alpha)\hat{k}_t &= \frac{\bar{N}}{1-\bar{N}}\hat{n}_t + \left[\chi + \frac{(1-\chi)\bar{C}^{1-\chi}}{\bar{C}^{1-\chi}+\phi\bar{m}^{1-\chi}}\right]\hat{c}_t + \left[\frac{(1-\chi)\phi\bar{m}^{1-\chi}}{\bar{C}^{1-\chi}+\phi\bar{m}^{1-\chi}}\right]\hat{m}_t \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + \hat{R}_t - E_t\hat{\pi}_{t+1} \\
\chi\hat{c}_t - \chi\hat{m}_t &= \left(\frac{\beta}{1+\mu-\beta}\right)\hat{R}_t \\
s_c\hat{c}_t + \frac{s_i}{\delta}\hat{k}_{t+1} &= a_t + \left((1-\alpha) + s_i\frac{1-\delta}{\delta}\right)\hat{k}_t + \alpha\hat{n}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \theta_t \\
\theta_t &= \rho^\theta\theta_{t-1} + \epsilon_t^\theta \\
a_t &= \rho^a a_{t-1} + \epsilon_t^a
\end{aligned}$$

Calibration We will adopt the same values as before: $\alpha = 0.58$, $\beta = 0.988$, $\delta = 0.025$, $\sigma = 1$, $\rho_a = 0.9$, $\sigma_a = 0.01$, $\rho^\theta = 0.481$, $\sigma^\theta = 0.009$ and $1+\mu = 1.015$. $\bar{A}$ is chosen such that the steady state level of output is unity. We chose η such that $\bar{N} = 0.20$. As before, ϕ is chosen in order to match the average M1 velocity of about $1/0.16$ and $\chi = 1/0.39$. The model is solved in **MIUmodel2.m** using the QZ decomposition. Figure 8 displays the impulse responses of the key macroeconomic variables to a 1% positive innovation in technology, and Figure 9 shows the impulse responses to a 1% positive innovation in money growth.

The impulse responses to a technology shock are very similar to those of the standard RBC model. This is despite the fact that there is no longer a classical dichotomy. Since real money balances affect the marginal utilities of consumption and leisure, they also affect the investment and labor decision. That money is no longer superneutral in the short run with nonseparable preferences is also clear from the fact that the real variables respond to the monetary shock. However, the real effects of a monetary expansion are *very* weak quantitatively. A monetary expansion reduces output and the response of investment has the opposite sign of the output response. As before, there is no dominant liquidity effect and inflation is not persistent after a monetary innovation.

You can verify that whether consumption, output and hours decrease or increase after a positive money growth shock is determined by the size of χ. If $\chi > 1$ and the interest rate elasticity is relatively low, then a decrease in m_t after a monetary expansion increases the

Figure 8: Response to a +1% technology shock

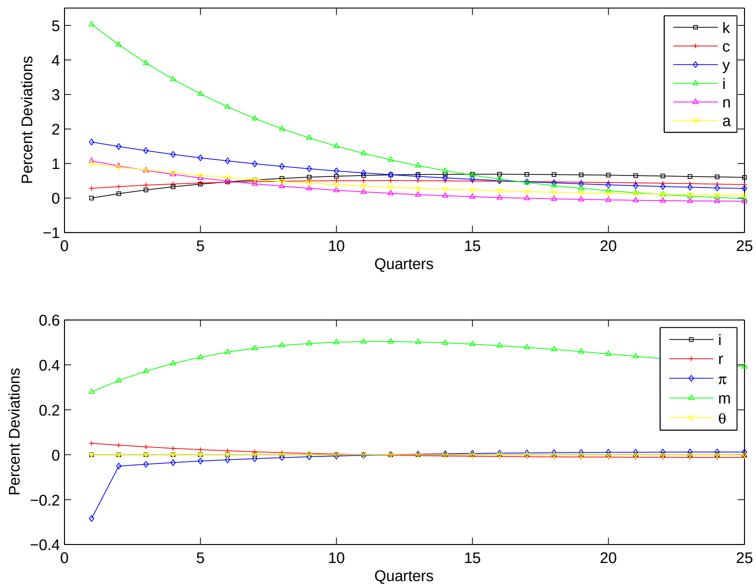
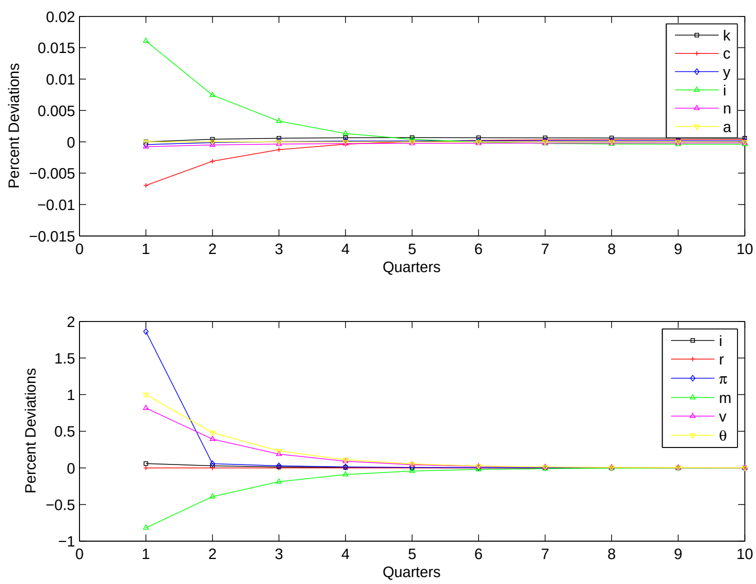


Figure 9: Response to a +1% money growth shock



marginal utility of leisure relative to consumption, which lowers labor supply and output. If $\chi < 1$ and the interest rate elasticity is relatively high, then a decrease in m_t after a monetary expansion lowers the marginal utility of leisure relative to consumption, which increases labor supply and output.

EXERCISE: Consider the environment above, but suppose now that the household's utility function is

$$u(C_t, m_t, 1 - N_t) = \frac{\left(C_t^{1-\chi} + \phi m_t^{1-\chi}\right)^{\frac{1-\sigma}{1-\chi}}}{1-\sigma} + \frac{\theta_l (1 - N_t)^{1-\eta}}{1-\eta}$$

1. Write down the equilibrium conditions.
2. Analyze the deterministic steady state and explain whether/why there is long-run nonsuperneutrality. Are these preferences consistent with balanced growth?
3. Loglinearize the equilibrium conditions and explain whether/why there is short-run nonsuperneutrality.
4. Calibrate the model assuming that $\sigma = 5$, $\chi = 1/0.39$ and $\eta = 5$. Analyze the dynamics after an innovation in money growth.
5. Verify that whether consumption, output and hours decrease after a positive money growth shock is determined by the sign of u_{cm} and explain.

2.2 A Cash-in-Advance (CIA) Model

This section incorporates an alternative way of modeling money by assuming a cash-in-advance (CIA) constraint. One example of this approach to modeling money is Cooley and Hansen (1989).

Environment The economy is populated by a representative household with preferences represented by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \ , \ 0 < \beta < 1 \tag{12}$$

where $C_t > 0$ is consumption in period t and L_t denotes time devoted to leisure. As in King et al. (1988), assume that the household has instantaneous utility:

$$u(C_t, L_t) = \begin{cases} \frac{(C_t v(L_t))^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log(C_t) + v(L_t) & \text{if } \sigma = 1 \end{cases} \tag{13}$$

and that the time constraint is $L_t = 1 - N_t$. The representative household enters every period t with nominal money balances M_{t-1}. In addition, these balances are augmented by a lump-sum money transfer by the government T_t. The key assumption in the CIA model is that households are required to acquire money balances to purchase goods intended for consumption. The household's consumption choice must satisfy the constraint:

$$P_t C_t \leq M_{t-1} + T_t$$

or equivalently

$$C_t \leq \frac{m_{t-1}}{1+\pi_t} + t_t \tag{14}$$

The households period t budget constraint is

$$C_t + I_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} \leq w_t N_t + r_t K_t + (1 + R_{t-1})\frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t}$$

or equivalently

$$C_t + I_t + b_t + m_t \leq w_t N_t + r_t K_t + \frac{1 + R_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} + t_t$$

and as before, the law of motion for the capital stock is

$$K_{t+1} = I_t + (1-\delta) K_t \ , \ 0 < \delta < 1$$

where I_t denotes gross investment. The household's problem is to choose the real quantities $\{C_t, b_t, m_t, N_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (12) subject to $b_t > -\bar{b}$, the law of motion for capital, the CIA constraints and the budget constraints and taking as given inflation and nominal interest rates $\{\pi_t, R_t\}_{t=0}^{\infty}$ as well as the real factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, the lumps sum transfers and the initial capital stock K_0 and real bond and money holdings and nominal interest rate b_{-1}, m_{-1}, R_{-1}. The behavior of the firms and the government as well as the definition of equilibrium are identical to the MIU model.

Money Demand In equilibrium, the following conditions must be satisfied in every period t at an *interior* solution, i.e. under the assumption that the CIA constraint is *always* binding:

$$\lambda_t - u_c(C_t, 1-N_t) + \Xi_t = 0 \tag{15a}$$

$$-\lambda_t + \beta E_t\left[\lambda_{t+1}\left((1-\alpha)A_{t+1}\left(\frac{K_{t+1}}{N_{t+1}}\right)^{-\alpha} + 1 - \delta\right)\right] = 0 \tag{15b}$$

$$-u_l(C_t, 1-N_t) + \lambda_t \alpha A_t \left(\frac{K_t}{N_t}\right)^{1-\alpha} = 0 \tag{15c}$$

$$-\lambda_t + \beta E_t\left[\lambda_{t+1}\frac{1+R_t}{1+\pi_{t+1}}\right] = 0 \tag{15d}$$

$$-\lambda_t + \beta E_t\left[u_c(C_{t+1}, 1-N_{t+1})\frac{1}{1+\pi_{t+1}}\right] = 0 \tag{15e}$$

$$C_t = m_t \tag{15f}$$

together with $C_t = A_t K_t^{1-\alpha} N_t^{\alpha} + (1-\delta)K_t - K_{t+1}$.

Equation (15a) links the marginal utility of wealth λ_t to the marginal utility of consumption. Note that both are no longer identical: the marginal utility of consumption exceeds λ_t by $\Xi_t > 0$, the value of liquidity services. Ξ_t is the multiplier associated with the CIA constraint. Equation (15b) is the familiar Euler equation describing the optimal consumption-investment choice, (15c) describes the optimal labor leisure trade-off, (15d) is the bond Euler equation, (15e) is the money demand equation, and (15f) is the CIA constraint. Note that (15a), (15d) and (15e) imply that $\Xi_{t+1} = 0$ whenever $R_t = 0$. A requirement for an interior solution in which the CIA constraint binds is that the nominal interest rate is strictly positive! The money demand equation reduces to

$$\beta E_t\left[\frac{\Xi_{t+1}}{1+\pi_{t+1}}\right] = \frac{R_t}{1+R_t}\lambda_t$$

which states that the marginal benefit of holding money (i.e. the expected liquidity services provided tomorrow) equals the opportunity cost $\frac{R_t}{1+R_t}$.

The Deterministic Steady State First note that unlike in the MIU model, the CIA constraint rules out speculative hyperinflations and $\bar{\pi} = \mu$. Second, the steady state values of capital, consumption, investment, labor input and output in the deterministic CIA model with elastic labor supply are different from those in the corresponding RBC model, and will depend on the money growth/inflation rate in the steady state. As in the extended MIU model, the deterministic CIA model with elastic labor supply displays long-run non-superneutrality. The key reason is that a binding CIA constraint and therefore $\bar{\Xi} > 0$ distorts the labor supply decision:

$$u_l(\bar{C}, 1-\bar{N}) < u_c(\bar{C}, 1-\bar{N})\bar{w} \tag{16}$$

since $\bar{\lambda} = \frac{\beta}{1+\mu} u_c(\bar{C}, 1-\bar{N}) < u_c(\bar{C}, 1-\bar{N})$. The higher the money growth rate μ, the lower the marginal value of the real wage, and therefore the lower the labor supply. Had we assumed an inelastic labor supply, the CIA model would be superneutral as in the basic MIU model. Finally, note that the requirement of a positive nominal interest rate requires

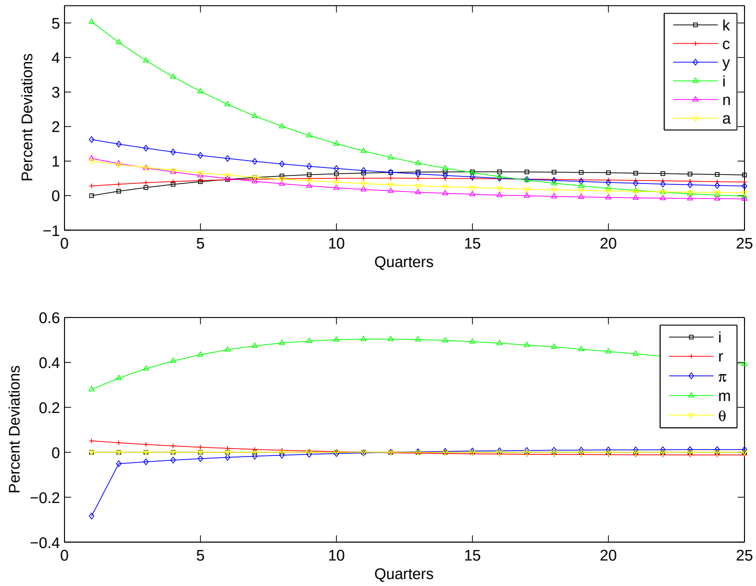
$$1 + \bar{R} = \frac{1+\mu}{\beta} > 1$$

Dynamics in the Stochastic Model The loglinearized dynamics around the deterministic steady state are described by

$$\begin{aligned}
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + (1-\beta(1-\delta))\, E_t(a_{t+1} - \alpha\hat{k}_{t+1} + \alpha\hat{n}_{t+1}) \\
a_t - (1-\alpha)\hat{n}_t + (1-\alpha)\hat{k}_t + \hat{\lambda}_t &= -\xi_{ll}\frac{\bar{N}}{1-\bar{N}}\hat{n}_t + \xi_{lc}\hat{c}_t \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + \hat{R}_t - E_t\hat{\pi}_{t+1} \\
\hat{\lambda}_t &= -\sigma E_t\hat{c}_{t+1} - \xi_{cl}\frac{\bar{N}}{1-\bar{N}}E_t\hat{n}_{t+1} - E_t\hat{\pi}_{t+1} \\
\hat{c}_t &= \hat{m}_t \\
s_c\hat{c}_t + \frac{s_i}{\delta}\hat{k}_{t+1} &= a_t + \left((1-\alpha) + s_i\frac{1-\delta}{\delta}\right)\hat{k}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \theta_t \\
\theta_t &= \rho^\theta\theta_{t-1} + \epsilon_t^\theta \\
a_t &= \rho^a a_{t-1} + \epsilon_t^a
\end{aligned}$$

Calibration We will adopt the same values as before: $\alpha = 0.58$, $\beta = 0.988$, $\delta = 0.025$, $\sigma = 1$, $\rho_a = 0.9$, $\sigma_a = 0.01$, $\rho^\theta = 0.481$, $\sigma^\theta = 0.009$ and $1+\mu = 1.015$. $\bar{A}$ is chosen such that

Figure 10: Response to a +1% technology shock



the steady state level of output is unity. We set $v(L) = \theta_l \log(L)$ and chose θ_l such that $\bar{N} = 0.20$. Our assumptions imply that $\xi_{cl} = \xi_{lc} = 0$ and $\xi_{ll} = -1$. The model is solved in **CIAmodel.m** using the QZ decomposition. Figure 10 displays the impulse responses of the key macroeconomic variables to a 1% positive innovation in technology, and Figure 11 shows the impulse responses to a 1% positive innovation in money growth.

Monetary shocks have a sizeable but opposite effect on consumption and investment, but only weak effects on output or hours. However, Cooley and Hansen (1989) show that the implications for the business cycle moments are only minor, see Figure 12.[4] Note that, unlike in the MIU model, the decrease in m_t after a monetary expansion always increases the marginal utility of consumption, increases the demand for leisure and therefore lowers the labor supply and output. As in the MIU model, there is no dominant liquidity effect and inflation is not very persistent after a monetary innovation.

[4]The results of Cooley and Hansen (1989) are not directly applicable to the model in the notes because of differences in calibration, most importantly of the value of the labor supply elasticity.

Figure 11: Response to a +1% money growth shock

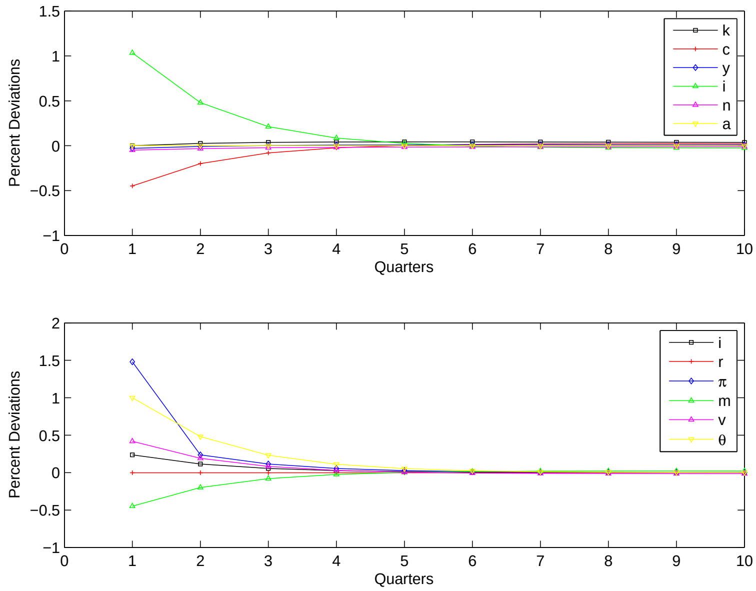


Figure 12: Source: Cooley and Hansen (1989)

TABLE 1—STANDARD DEVIATIONS IN PERCENT AND CORRELATIONS WITH OUTPUT FOR U.S. AND ARTIFICIAL ECONOMICS

Series	Quarterly U.S. Time Series[a] (1955.3–1984.1) Standard Deviation	Quarterly U.S. Time Series[a] (1955.3–1984.1) Correlation with Output	Economy with Constant Growth Rate ($\bar{g}$ = 0.99–1.15)[b] Standard Deviation	Economy with Constant Growth Rate ($\bar{g}$ = 0.99–1.15)[b] Correlation with Output
Output	1.74	1.00	1.76 (0.22)	1.00 (0.00)
Consumption	0.81	0.65	0.51 (0.07)	0.87 (0.02)
Investment	8.45	0.91	5.71 (0.74)	0.99 (0.00)
Capital Stock	0.38	0.28	0.48 (0.09)	0.07 (0.07)
Hours	1.41	0.86	1.34 (0.18)	0.98 (0.00)
Productivity	0.89	0.59	0.51 (0.07)	0.87 (0.03)
Price Level: CPI	1.59	−0.48	0.51 (0.07)	−0.87 (0.02)
Price Level: GNP Deflator	0.98	−0.53		

Series	Economy with Autoregressive Growth Rate ($\bar{g}$ = 1.015)[b] Standard Deviation	Economy with Autoregressive Growth Rate ($\bar{g}$ = 1.015)[b] Correlation with Output	Economy with Autoregressive Growth Rate ($\bar{g}$ = 1.15)[b] Standard Deviation	Economy with Autoregressive Growth Rate ($\bar{g}$ = 1.15)[b] Correlation with Output
Output	1.73 (0.22)	1.00 (0.00)	1.74 (0.22)	1.00 (0.00)
Consumption	0.62 (0.07)	0.72 (0.07)	0.65 (0.07)	0.70 (0.05)
Investment	5.69 (0.76)	0.97 (0.01)	5.69 (0.77)	0.97 (0.01)
Capital Stock	0.48 (0.10)	0.06 (0.07)	0.48 (0.10)	0.06 (0.06)
Hours	1.33 (0.17)	0.98 (0.01)	1.33 (0.17)	0.98 (0.01)
Productivity	0.50 (0.07)	0.87 (0.03)	0.50 (0.07)	0.87 (0.03)
Price Level	1.70 (0.34)	−0.27 (0.16)	1.93 (0.27)	−0.25 (0.16)

2.3 A Shopping Time (ST) Model

Shopping time models assume that the purchase of goods requires the input of transaction services which in turn are provided by money and time. One example of this approach to modeling money is King and Plosser (1984).

Environment As before, the economy is populated by a representative household with preferences represented by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \ , \ 0 < \beta < 1$$

where $C_t > 0$ is consumption in period t and L_t denotes time devoted to leisure. The instantaneous utility function is again given by (13). The key assumption in the shopping time model is that the household's time constraint is

$$L_t + N_t + S_t = 1$$

The unit time endowment consists of time devoted to leisure L_t, work N_t and shopping time S_t. The amount of shopping time S_t needed to purchase a particular level of consumption C_t is assumed to be negatively related to the household's holdings of real money balances m_t.[5] The shopping or transaction technology is given by

$$S_t = \Phi\left(C_t, m_t\right) \tag{18}$$

where Φ, Φ_c, Φ_{cc}, $\Phi_{mm} \geq 0$ and Φ_m, $\Phi_{cm} \leq 0$. The households period t budget constraint is

$$C_t + I_t + b_t + m_t \leq w_t N_t + r_t K_t + \frac{1 + R_{t-1}}{1 + \pi_t} b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + t_t$$

and the law of motion for the capital stock is

$$K_{t+1} = I_t + (1 - \delta) K_t \ , \ 0 < \delta < 1$$

where I_t denotes gross investment. The household's problem is to choose the real quantities $\{C_t, b_t, m_t, N_t, S_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize (2.3) subject to $b_t > -\bar{b}$, the law of motion for capital, the CIA constraints and the budget constraints and taking as given inflation and

[5] Note that we assume that only the purchase of *consumption* goods requires the input of transaction services. It is straightforward to impose the same requirement to the purchase of goods for *investment*.

nominal interest rates $\{\pi_t, R_t\}_{t=0}^{\infty}$ as well as the real factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, the lump sum transfers and the initial capital stock K_0 and real bond and money holdings and nominal interest rate b_{-1}, m_{-1}, R_{-1}. The behavior of the firms and the government as well as the definition of equilibrium are identical to the MIU and CIA model.

Money Demand The first order necessary conditions are now

$$\lambda_t - u_c(C_t, L_t) + u_l(C_t, L_t)\Phi_c(C_t, m_t) = 0 \tag{19a}$$

$$-\lambda_t + \beta E_t\left[\lambda_{t+1}\left((1-\alpha)A_{t+1}\left(\frac{K_{t+1}}{N_{t+1}}\right)^{-\alpha} + 1 - \delta\right)\right] = 0 \tag{19b}$$

$$-u_l(C_t, L_t) + \lambda_t \alpha A_t \left(\frac{K_t}{N_t}\right)^{1-\alpha} = 0 \tag{19c}$$

$$-\lambda_t + \beta E_t\left[\lambda_{t+1}\frac{1+R_t}{1+\pi_{t+1}}\right] = 0 \tag{19d}$$

$$-\lambda_t - u_l(C_t, L_t)\Phi_m(C_t, m_t) + \beta E_t\left[\lambda_{t+1}\frac{1}{1+\pi_{t+1}}\right] = 0 \tag{19e}$$

$$L_t + N_t + \Phi\left(C_t, m_t\right) = 1 \tag{19f}$$

together with $C_t = A_t K_t^{1-\alpha} N_t^{\alpha} + (1-\delta)K_t - K_{t+1}$.

Equation (19a) links the marginal utility of wealth λ_t to the marginal utility of consumption. Again, both are not identical: the marginal utility of consumption exceeds λ_t by $u_l(C_t, L_t)\Phi_c(C_t, m_t) > 0$, the value of transaction services. Equation (19b) is the familiar Euler equation describing the optimal consumption-investment choice, (19c) describes the optimal labor leisure trade-off, (19d) is the bond Euler equation, (19e) is the money demand equation, and (19f) is the time constraint.

The money demand equation reduces to

$$-\frac{u_l(C_t, L_t)\Phi_m(C_t, m_t)}{\lambda_t} = \frac{R_t}{1+R_t}$$

Using the first-order condition for the labor leisure choice, we can write money demand as

$$-w_t \Phi_m(C_t, m_t) = \frac{R_t}{1+R_t}$$

which states that the marginal benefit of holding money (i.e. the time saved valued at the real wage) equals the opportunity cost $\frac{R_t}{1+R_t}$.

The Deterministic Steady State We can define a function $\tilde{u}(C_t, m_t, L_t) = u(C_t, 1 - N_t - \Phi(C_t, m_t))$ that gives utility as a function of consumption, leisure and real money holdings. Thus the shopping time model can motivate the presence of m_t in the utility function, such that the analysis from the MIU model applies.

Dynamics in the Stochastic Model The loglinearized dynamics around the deterministic steady state are now described by

$$\begin{aligned}
\hat{\lambda}_t - (1-b)\omega_{cm}\hat{m}_t &= (-b\sigma + (1-b)(\xi_{lc} + \omega_{cc}))\,\hat{c}_t + (b\xi_{cl} + (1-b)\xi_{ll})\,\hat{l}_t \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + (1 - \beta(1-\delta))\,E_t(a_{t+1} - \alpha\hat{k}_{t+1} + \alpha\hat{n}_{t+1}) \\
a_t - (1-\alpha)\hat{n}_t + (1-\alpha)\hat{k}_t + \hat{\lambda}_t &= \xi_{ll}\hat{l}_t + \xi_{lc}\hat{c}_t \\
\hat{\lambda}_t &= E_t\hat{\lambda}_{t+1} + \hat{R}_t - E_t\hat{\pi}_{t+1} \\
a_t - (1-\alpha)\hat{n}_t + (1-\alpha)\hat{k}_t + \omega_{mc}\hat{c}_t + \omega_{mm}\hat{m}_t &= \frac{\beta}{1+\mu-\beta}\hat{R}_t \\
\hat{l}_t + \frac{\bar{N}}{\bar{L}}\hat{n}_t &= -\frac{\Phi(\bar{C},\bar{m})}{\bar{L}}\left(\phi_c\hat{c}_t + \phi_m\hat{m}_t\right) \\
s_c\hat{c}_t + \frac{s_i}{\delta}\hat{k}_{t+1} &= a_t + \left((1-\alpha) + s_i\frac{1-\delta}{\delta}\right)\hat{k}_t \\
\hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \theta_t \\
\theta_t &= \rho^\theta\theta_{t-1} + \epsilon_t^\theta \\
a_t &= \rho^a a_{t-1} + \epsilon_t^a
\end{aligned}$$

where $b = \frac{u_c(\bar{C},\bar{L})}{\bar{\lambda}}$, ω_{ab} is the elasticity of Φ_a with respect to b and ϕ_a is the elasticity of Φ with respect to a.

Calibration We will assume the following functional form for the transaction technology

$$\Phi(C_t, m_t) = \tau\frac{C_t}{1+m_t}\ ,\ \tau > 0$$

This functional form implies that $\omega_{cc} = 0$, $\omega_{cm} = -\frac{\bar{m}}{1+\bar{m}}$, $\omega_{mc} = 1$, $\omega_{mm} = -\frac{2\bar{m}}{1+\bar{m}}$, $\phi_c = 1$ and $\phi_m = -\frac{\bar{m}}{1+\bar{m}}$. For the parameters, we take the same values as before: $\alpha = 0.58$, $\beta = 0.988$, $\delta = 0.025$, $\sigma = 1$, $\rho_a = 0.9$, $\sigma_a = 0.01$, $\rho^\theta = 0.481$, $\sigma^\theta = 0.009$ and $1+\mu = 1.015$. $\bar{A}$ is chosen such that the steady state level of output is unity. We set $v(L) = \theta_l \log(L)$ and chose θ_l such that $\bar{N} = 0.20$. τ is chosen to match the M1 money velocity of about $1/0.16$. Our assumptions on the preferences imply that $\xi_{cl} = \xi_{lc} = 0$ and $\xi_{ll} = -1$.

The model is solved in **STmodel.m** using the QZ decomposition. Figure 13 displays the impulse responses of the key macroeconomic variables to a 1% positive innovation in technology, and Figure 14 shows the impulse responses to a 1% positive innovation in money growth. The results are very similar to the earlier extended MIU model.

Figure 13: Response to a +1% technology shock

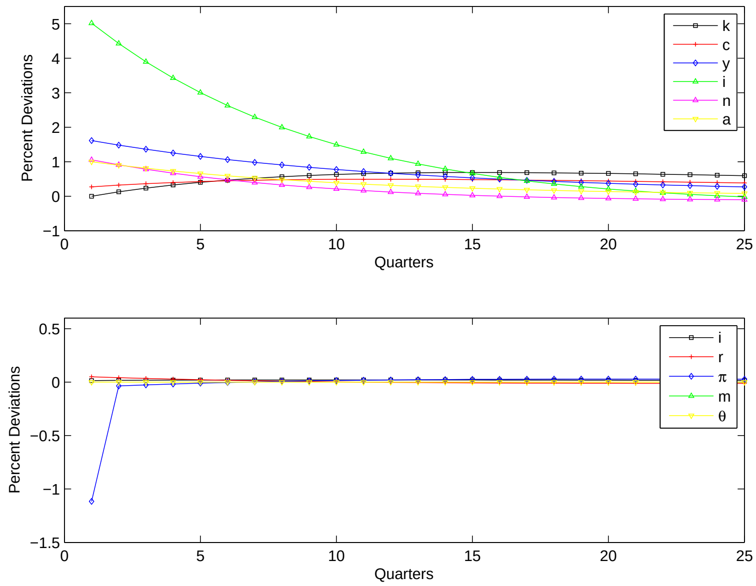


Figure 14: Response to a +1% money growth shock