Models with Nominal Rigidities

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Introduction

In the chapter on flexible price monetary models, we saw that the MIU, CIA and ST models can generate real effects to monetary disturbances. However, these effects were qualitatively inconsistent with the empirical evidence produced for instance by monetary structural VARs. In most cases, the responses of the real variables in realistic simulations were also quantitatively so insignificant that money basically did not matter in practice. This chapter presents some alternative models in which there are significant and empirically plausible short-run effects of monetary disturbances. These models are important tools for understanding how monetary policy and other aggregate demand shocks affect economic behavior and the business cycle.
There are generally three classes of models that are relatively successful in reconciling the long-run neutrality of money with the apparent short-run nonneutrality: the first two are models of *imperfect information* and models of *limited participation*, both of which maintain the assumption of flexible prices. The third, and arguably most popular, class of models incorporate *nominal rigidities*, either in the form of sticky goods prices or sticky nominal wages. This chapter will focus exclusively on sticky price models.

In the next section we will take a very simple dynamic general equilibrium model and introduce monopolistic composition, an essential ingredient for most sticky price model. The second section introduces the simplest form of price stickiness: one period preset prices. The third section introduces a more realistic model of staggered price setting. The fourth section discusses a benchmark model that can be used for the analysis of monetary policy. The final section discusses the microeconomic evidence for nominal price rigidities.

1 Monopolistic Competition

This section extends a basic MIU model by allowing for monopolistic competition in the goods market. Introducing some degree of monopoly power is essential to allow firms to set their own prices. For simplicity, the model abstracts from capital accumulation. You can think of capital being used in production, but capital goods are in fixed supply, do not depreciate and their allocation across households (or firms) cannot be changed.

Households The economy is populated by a representative household with preferences represented by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t, m_t) \ , \ 0 < \beta < 1 \tag{1}$$

where N_t is labor and $m_t = \frac{M_t}{P_t}$ is the real value of money holdings, $M_t > 0$ denotes nominal money balances. There is a continuum of consumption goods distributed uniformly over the unit interval. C_t is a *composite consumption index* given by the Dixit-Stiglitz aggregator

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \ , \ \theta > 1$$

where $C_t(i)$ is the quantity of good $i \in [0, 1]$ consumed in period t and θ is the elasticity of substitution among consumption goods. P_t is the corresponding utility-based price index of the prices of all goods. It is defined as the minimum amount of nominal spending required to obtain (at least) one unit of C_t. In the expenditure minimization problem:

$$\min_{c_t(i)} \int_0^1 P_t(i) C_t(i) di \text{ s.t. } C_t \geq 1$$

P_t is the Lagrange multiplier on the constraint. The first order condition to this problem is

$$P_t(i) = P_t \left(\frac{C_t(i)}{C_t} \right)^{-\frac{1}{\theta}} \tag{2a}$$

$$\Leftrightarrow C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} C_t \tag{2b}$$

together with $C_t = 1$. Substituting (2a) into the objective function, we have

$$\begin{aligned} \int_0^1 P_t(i)C_t(i)di &= P_t\left(\int_0^1 C_t(i)^{\frac{\theta-1}{\theta}}di\right)C_t^{\frac{1}{\theta}} \\ &= P_t C_t^{\frac{\theta-1}{\theta}} C_t^{\frac{1}{\theta}} \\ &= P_t C_t \\ &= P_t \end{aligned}$$

Substituting (2b) into the objective function, we have that

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta}di\right)^{\frac{1}{1-\theta}}$$

which is called the *Dixit-Stiglitz price index*. It implies that total nominal consumption expenditure equals P_tC_t. The household's period budget constraint is:

$$\int_0^1 P_t(i)C_t(i)di + B_t + M_t \leq W_tN_t + M_{t-1} + (1+R_{t-1})B_{t-1} + T_t + Q_t$$

where $P_t(i)$ is the nominal price of good i, W_t is the nominal wage, T_t are lump-sum monetary transfers by the government, B_t are government issued bonds and Q_t are nominal profits from firm ownership. Utility maximization implies that the households will solve the expenditure minimization problem above to decide on the optimal *intratemporal* allocation of the different consumption goods. The budget constraint relevant for the *intertemporal* problem can be reduced to

$$P_tC_t + B_t + M_t \leq W_tN_t + M_{t-1} + (1+R_{t-1})B_{t-1} + T_t + Q_t$$

The household's intertemporal problem is to choose the real quantities $\{C_t, m_t, N_t\}_{t=0}^{\infty}$ to maximize (1) subject to the budget constraints for a given level of initial money holdings and taking as given prices, monetary transfers and firm profits.

As seen previously, the household's first order conditions lead to

$$\begin{aligned} u_c(C_t, m_t, 1-N_t) &= \beta E_t\left[u_c(C_{t+1}, m_{t+1}, 1-N_{t+1})\frac{1+R_t}{1+\pi_{t+1}}\right] \\ \frac{u_m(C_t, m_t, 1-N_t)}{u_c(C_t, m_t, 1-N_t)} &= \frac{R_t}{1+R_t} \\ u_c(C_t, m_t, 1-N_t)w_t &= u_l(C_t, m_t, 1-N_t) \end{aligned}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage.

Firms There is a continuum of firms in the economy that are uniformly distributed over the unit interval. Each firm is indexed by $i \in [0, 1]$ and produces a differentiated good using the technology

$$Y_t(i) = A_t N_t^{\alpha}(i) \ , \ 0 < \alpha < 1, A > 0 \tag{3}$$

$Y_t(i)$ is output of firm i and $N_t(i)$ is the quantity of labor used by firm i. A_t is a random stationary productivity process common to all firms. Labor input is rented in a competitive market at a real wage w_t. Each firm acknowledges how the demand for its differentiated good i depends on its own price level $P_t(i)$

$$Y_t(i) = C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} C_t \tag{4}$$

At the same time, the individual firm regards itself as unable to affect the evolution of the variables C_t and P_t and takes these as given.

As there is no intertemporal dimension to the firm's problem, the optimal $P_t(i)$ will maximize period t profits

$$\begin{aligned}\frac{Q_t(i)}{P_t} &= \frac{P_t(i)}{P_t} Y_t(i) - w_t N_t(i) \\ &= \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} C_t - \frac{w_t}{A_t^{\frac{1}{\alpha}}} \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\theta}{\alpha}} C_t^{\frac{1}{\alpha}}\end{aligned}$$

Optimal price setting requires

$$\begin{aligned}(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{C_t}{P_t} + \frac{w_t}{A_t^{\frac{1}{\alpha}}} \frac{\theta}{\alpha} \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\alpha}-1} \frac{C_t^{\frac{1}{\alpha}}}{P_t} &= 0 \\ \Leftrightarrow (1-\theta) Y_t(i) + \frac{\theta}{\alpha} w_t N_t(i) \frac{P_t}{P_t(i)} &= 0 \\ \Leftrightarrow (1-\theta) P_t(i) Y_t(i) + \frac{\theta}{\alpha} W_t N_t(i) &= 0 \\ \Leftrightarrow P_t(i) = \frac{\theta}{\theta - 1} \frac{W_t N_t(i)}{\alpha Y_t(i)} &\end{aligned}$$

The optimal price setting condition implies a fixed markup over the marginal cost of production, i.e.

$$P_t(i) \quad = \quad \omega MC_t(i)$$

where $\omega = \frac{\theta}{\theta-1} > 1$ is the constant markup and $MC_t(i) = \frac{W_t N_t(i)}{\alpha Y_t(i)}$ is the marginal cost of production. Note that as $\theta \to \infty$, the goods become perfect substitutes and the markup goes to one. $MC_t(i)$ is defined as the Lagrange multiplier in the cost minimization problem

$$\min_{N_t(i)} W_t N_t(i) \text{ s.t. } Y_t(i) \geq \bar{Y}$$

The first order condition is

$$\begin{aligned} W_t &= MC_t(i)\alpha A_t(N_t(i))^{\alpha-1} \\ \Leftrightarrow MC_t(i) &= \frac{W_t N_t(i)}{\alpha Y_t(i)} \end{aligned}$$

Government The government is the monopoly supplier of money, which it uses to finance the lump-sum transfers and supplies a fixed amount of nominal bonds. Assume that the growth rate of the money supply in deviation of the steady state growth rate, denoted by $\theta_t = \frac{M_t}{M_{t-1}} - \mu - 1$ is an stationary mean zero exogenous random variable and μ is the average money growth rate.

Equilibrium In a *symmetric equilibrium*, all firms maximize profits and choose identical prices ($P_t(i) = P_t$, $\forall i$) and labor input levels ($N_t(i) = N_t$, $\forall i$), households solve their utility maximization problem and all markets (for goods, assets and labor) clear.

Model Dynamics We will assume the following simple functional form for the instantaneous utility function:

$$u(C_t, 1 - N_t, m_t) = \tilde{u}(C_t, N_t, m_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \phi_m \frac{m_t^{1-\chi}}{1-\chi} - \phi_n \frac{N_t^{1+\xi}}{1+\xi} \ , \sigma, \chi, \xi, \phi_m, \phi_n > 0$$

The dynamics of aggregate consumption C_t, hours N_t, output Y_t and real money balances m_t can be summarized by the following conditions:

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1+R_t}{1+\pi_{t+1}} \right] \tag{5a}$$

$$Y_t = C_t \tag{5b}$$

$$\phi_m m_t^{-\chi} = \frac{R_t}{1+R_t} C_t^{-\sigma} \tag{5c}$$

$$\frac{m_t}{m_{t-1}}(1+\pi_t) = 1+\mu+\theta_t \tag{5d}$$

$$\phi_n N_t^{\xi} = \frac{\alpha}{\omega} \frac{Y_t}{N_t} C_t^{-\sigma} \tag{5e}$$

$$Y_t = A_t N_t^{\alpha} \tag{5f}$$

Equation (5a) and (5b) can be combined to derive an *IS-relationship* between output and the interest rate that represents goods market clearing:

$$Y_t^{-\sigma} = \beta E_t \left[Y_{t+1}^{-\sigma} \frac{1+R_t}{1+\pi_{t+1}} \right] \tag{6a}$$

Equation (5b), (5c) and (5d) can be combined to derive an *LM-relationship* between output and the interest rate that represents asset market clearing:

$$\frac{R_t}{1+R_t} = \phi_m \left(\frac{1+\mu+\theta_t}{1+\pi_t} m_{t-1} \right)^{-\chi} Y_t^{\sigma} \tag{6b}$$

Finally, combine (5e) and (5f) to get aggregate supply AS:

$$Y_t = A_t^{\frac{1+\xi}{1+\xi+\alpha(\sigma-1)}} \left(\frac{\alpha}{\phi_n \omega} \right)^{\frac{\alpha}{1+\xi+\alpha(\sigma-1)}} \tag{6c}$$

Equation (6a) to (6c) constitute a very tractable flexible price macroeconomic model with one aggregate supply shock A_t and one aggregate demand shock θ_t.

Consider the following loglinear approximations:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) \tag{IS}$$

$$\sigma \hat{y}_t = \chi(\theta_t + \hat{m}_{t-1} - \hat{\pi}_t) + \frac{\beta}{1+\mu-\beta} \hat{R}_t \tag{LM}$$

$$\hat{y}_t = \frac{1+\xi}{1+\xi+\alpha(\sigma-1)} a_t \tag{AS}$$

Figure 1: Flexible Price Model: i.i.d. shocks

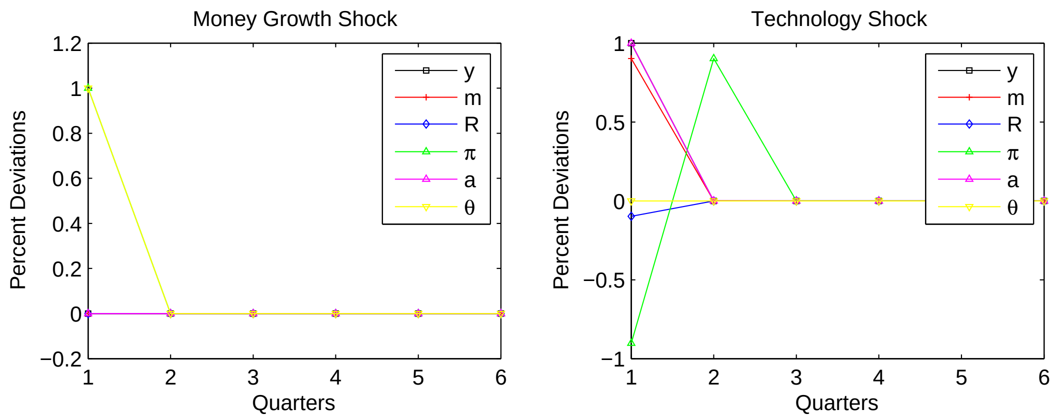
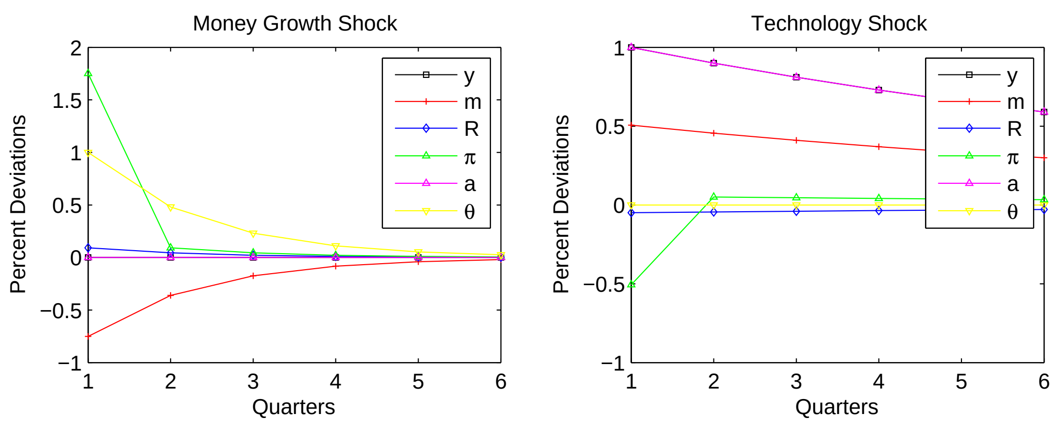


Figure 2: Flexible Price Model: persistent shocks



Consider the following simple parametrization of the model: $\alpha = 0.58$, $\beta = 0.988$, $\sigma = 1$, $\xi = 1$, $\chi = 10$, $\mu = 0$. At this point, it is not necessary to specify values for the other parameters as they do not influence the model dynamics. Note for instance that the value of θ, the elasticity of substitution among goods, only affects the long run level of output but not the short-run dynamics. To improve our understanding of the model, Figure 1 plots the impulse responses to a money growth and technology shock under the assumption that both are i.i.d. random variables. First consider the money growth shock: the model displays monetary neutrality and superneutrality in both the short and long run and the AS curve is perfectly vertical, i.e. we obtain the classical dichotomy. This implies that AD-shocks such as an increase in the money growth rate cannot change output which is always at its long run natural level given by equation (6c). Because the money shock is i.i.d., there is no change in expected inflation or the nominal interest rate and prices move one for one with the expanded money supply.

In response to a positive innovation in productivity, the natural output level increases, the AS curve shifts out and the price level drops. In the period after the shock the AS curve returns to its initial position and so does the price level. As a result of the decline in the price level in the period of the shock the LM curve shifts out, real money balances increase and the nominal interest rate declines. After the shock, the price level goes back up and the LM curve returns to its initial position.

Figure 2 plots the impulse responses to a money growth and technology shock under the assumption that both shocks are persistent. As in the previous chapters, both follow AR(1) processes with the persistence of θ equal to $\rho_\theta = 0.48$ and the persistence of the technology shock equal to $\rho_a = 0.9$. After a money growth shock, the additional effects that come into play are because of a change in expected inflation. The drop in real money demand implies that the price level must increase more than proportional to the money supply, such that real money balances drop. The nominal interest rate increases: there is no (dominating) liquidity effect.

After a persistent increase in productivity, the price level is slow to return to its initial level as the natural level output is now persistently higher. The effects of higher expected inflation that reduce demand for real money balances (LM curve shifts to the right) together with the effect of future higher expected output on consumption (IS curve shifts to the right) make the initial drop in the price level smaller than in the i.i.d case.

The Matlab-program used to compute these impulse responses is **stickyprice1.m**.

2 One-Period Sticky Prices

Now suppose the firms must set their prices one period in advance, such that period t prices do not react to the realizations of the period t demand and supply shocks. Assume that firms are committed to supply whatever quantity buyers may wish to purchase at the predetermined price, and hence firms obtain whatever labor input is necessary to fill orders. The optimal $P_t(i)$ which is set in period $t-1$ maximizes the discounted value of expected profits

$$
\begin{aligned}
E_{t-1}\left[\beta\frac{\lambda_t}{\lambda_{t-1}}\frac{Q_t}{P_t}\right] &= E_{t-1}\left[\beta\frac{\lambda_t}{\lambda_{t-1}}\left(\frac{P_t(i)}{P_t}Y_t(i)-w_tN_t(i)\right)\right] \\
&= E_{t-1}\left[\beta\frac{\lambda_t}{\lambda_{t-1}}\left(\left(\frac{P_t(i)}{P_t}\right)^{1-\theta}C_t-\frac{w_t}{A_t^{\frac{1}{\alpha}}}\left(\frac{P_t(i)}{P_t}\right)^{\frac{-\theta}{\alpha}}C_t^{\frac{1}{\alpha}}\right)\right]
\end{aligned}
$$

Optimal price setting requires

$$
\begin{aligned}
E_{t-1}\left[\lambda_t\left((1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\frac{C_t}{P_t}+\frac{w_t}{A_t^{\frac{1}{\alpha}}}\frac{\theta}{\alpha}\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\alpha}-1}\frac{C_t^{\frac{1}{\alpha}}}{P_t}\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[\lambda_t\left((1-\theta)\frac{P_t(i)}{P_t}Y_t(i)+\frac{\theta}{\alpha}w_tN_t(i)\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[\lambda_t\left(\frac{P_t(i)}{P_t}Y_t(i)-\frac{\omega}{\alpha}w_tN_t(i)\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[\lambda_tY_t(i)\left(\frac{P_t(i)}{P_t}-\omega\frac{MC_t(i)}{P_t}\right)\right] &= 0
\end{aligned}
$$

In a symmetric equilibrium in which all firms set identical prices and chose the same number of hours, the optimal price setting condition reduces to

$$
\begin{aligned}
\Leftrightarrow E_{t-1}\left[Y_t^{1-\sigma}\left(1-\omega\frac{MC_t}{P_t}\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[\left(Y_t^{1-\sigma}-\frac{\omega\phi_n}{\alpha}N_t^{1+\xi}\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[\left(Y_t^{1-\sigma}-\frac{\omega\phi_n}{\alpha}A_t^{-\frac{1+\xi}{\alpha}}Y_t^{\frac{1+\xi}{\alpha}}\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[Y_t^{1-\sigma}\left(1-\frac{\omega\phi_n}{\alpha}A_t^{-\frac{1+\xi}{\alpha}}Y_t^{\frac{1+\xi+\alpha(\sigma-1)}{\alpha}}\right)\right] &= 0 \\
\Leftrightarrow E_{t-1}\left[Y_t^{1-\sigma}\left(1-\left(\frac{Y_t}{Y_t^n}\right)^{\frac{1+\xi+\alpha(\sigma-1)}{\alpha}}\right)\right] &= 0 \qquad (7)
\end{aligned}
$$

where Y_t^n is defined as the *natural level of output* which equals the level of output that would prevail in the flexible price economy, i.e.

$$
Y_t^n = A_t^{\frac{1+\xi}{1+\xi+\alpha(\sigma-1)}}\left(\frac{\alpha}{\phi_n\omega}\right)^{\frac{\alpha}{1+\xi+\alpha(\sigma-1)}} \qquad (8)
$$

It is appropriate to think of (7) as the *short-run aggregate supply curve* and (8) as the *long run aggregate supply curve*. Loglinearizing yields the following aggregate supply relationships:

$$E_{t-1}\left[\hat{y}_t - \hat{y}_t^n\right] = 0 \tag{SRAS}$$

$$\hat{y}_t^n = \frac{1+\xi}{1+\xi+\alpha(\sigma-1)} a_t \tag{LRAS}$$

where $\hat{y}_t - \hat{y}_t^n$ is called the *output gap*. The SRAS and LRAS, together with the IS-LM equations of the previous section determine the dynamics of the endogenous variables. Note that the labor supply equation (and the value of the wage elasticity!) is irrelevant in determining the short-run equilibrium level of output. Any unanticipated movements in aggregate demand are met by varying the nominal wage such as to supply the required level of goods: the SRAS aggregate supply curve is horizontal at the preset price level and output may deviate from the natural level. In the short run, the level of output is determined by aggregate demand only. As a result, in the sticky price model, money will neither be neutral nor superneutral in the short run.

Consider the same parametrization of the model as before: $\alpha = 0.58$, $\beta = 0.988$, $\sigma = 1$, $\xi = 1$, $\chi = 10$ and $\mu = 0$. Again, it is not necessary to specify values for any other parameters since they do not influence the model dynamics. Note that the steady state and the natural level of output are identical to the flexible price economy from the last section. Figure 3 plots the impulse responses to a money growth and technology shock in the sticky price model for the case where the shocks are i.i.d. Contrary to the flexible price model, money is no longer neutral: output expands after a positive money growth shock. At fixed prices, equilibrium in the market for money now requires a lower nominal interest rate to equate money supply to money demand: the LM curve shifts to the right and so does the AD curve. Note that the sticky price model delivers a liquidity effect. Firms cannot change prices and meet the increased demand by increasing the nominal wage, labor input and production. The implied drop in profits constitutes another reason why it was necessary to introduce monopolistic competition and excess profits. In the period after the shock, prices readjust, the LM curve and AD curves shift back to their original positions and output is back at its natural level.

In response to a technology shock, there is no change in output, nor in either of the other variables plotted. Since there is no change in aggregate demand, firms supply the quantity demanded at current prices. As a result of the productivity improvement, they will be able

Figure 3: One-Period Sticky Price Model: i.i.d. shocks

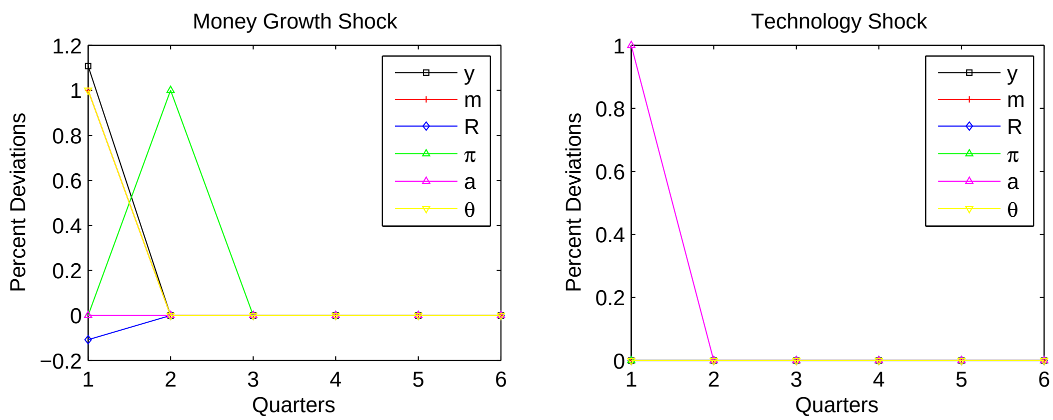
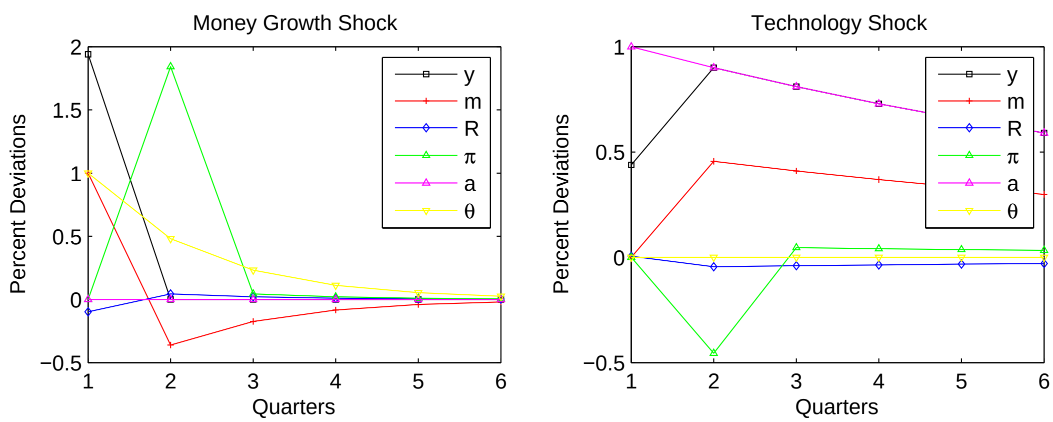


Figure 4: One-Period Sticky Price Model: persistent shocks



to produce the same quantity with less labor input, so the nominal wage drops and hours worked decrease.

Figure 3 plots the impulse responses to a money growth and technology shock in the sticky price model for the case where the shocks are persistent: as before, $\rho_\theta = 0.48$ and $\rho_a = 0.9$. In response to the money growth shock, there is a drop in demand for real money balances due to higher expected inflation. This additional effect further shifts out the LM curve and the liquidity effect is larger. As a results, the increase in aggregate demand is larger if the shock is persistent, and output expands more. The real effects are very transitory however: the period after the shock the price adjust and output returns to its natural level. From the second period onwards, the dynamics are identical to the case of flexible prices.

In response to a persistent productivity improvement, lower expected inflation shifts the LM curve to the left and reduces aggregate demand, but higher future income shifts the is curve to the right and increases aggregate demand. The real interest rate must unambiguously increase, but there are offsetting effects on aggregate demand. It turns out that the future income effect dominates such that the AD curve shifts the right in the period of the shock. After one period prices adjust and the dynamics are the same as in the flexible price case. The Matlab-program used to compute these impulse responses is **stickyprice2.m**.

3 Staggered Price Setting: The Calvo Model

This section presents a version of a model with staggered price setting introduced by Calvo (1983). The section is based on Chapter 3 of Woodford (2003) and Chapter 5.4 of Walsh (2003).

Staggered pricesetting In the Calvo model, a fraction γ of goods prices remain unchanged each period, whereas a fraction $1-\gamma$ of firms chose new prices for their goods. For simplicity, the probability that any given price can adjust is $1-\gamma$ and is independent of the length of time since the price was last adjusted. Each supplier that chooses a new price for its good in period t faces exactly the same decision problem and the optimal price P_t^* chosen is the same for all of them in equilibrium. The remaining fraction γ of prices are a subset of the prices charged in $t-1$. Given the assumption that every price has the same probability of being changed, each price appears in the period t distribution of unchanged prices with the same relative frequency as in the period $t-1$ distribution of prices. This allows us to write the Dixit Stiglitz price index as

$$\begin{aligned} P_t &= \left((1-\gamma)(P_t^*)^{1-\theta} + \gamma \int_0^1 P_{t-1}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}} \\ &= \left((1-\gamma)(P_t^*)^{1-\theta} + \gamma P_{t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}} \end{aligned} \tag{9}$$

So in order to find P_t it suffices to know the new optimal price P_t^* and the price level in period $t-1$, P_{t-1}.

A firm that changes its price in period t maximizes expected discounted profits

$$\begin{aligned} & E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\frac{\lambda_s}{\lambda_t}\left(\frac{P_t(i)}{P_s}Y_s(i) - w_s N_s(i)\right)\right] \\ = \; & E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\frac{\lambda_s}{\lambda_t}\left(\left(\frac{P_t(i)}{P_s}\right)^{1-\theta} C_s - \frac{w_s}{A_s^{\frac{1}{\alpha}}}\left(\frac{P_t(i)}{P_s}\right)^{\frac{-\theta}{\alpha}} C_s^{\frac{1}{\alpha}}\right)\right] \end{aligned}$$

Optimal price-setting requires

$$\begin{aligned} E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\frac{\lambda_s}{\lambda_t}\left((1-\theta)\frac{P_t(i)}{P_s}Y_s(i) + \frac{\theta}{\alpha}w_s N_s(i)\right)\right] &= 0 \\ \Leftrightarrow E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\frac{\lambda_s}{\lambda_t}Y_s(i)\left(\frac{P_t(i)}{P_s} - \omega\frac{MC_s(i)}{P_s}\right)\right] &= 0 \end{aligned}$$

In equilibrium $P_t(i) = P_t^*$ such that

$$E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\lambda_s Y_s^t\left(\frac{P_t^*}{P_s} - \omega\frac{MC_s^t}{P_s}\right)\right] \;\; = \;\; 0$$

where $Y_s^t = \left(\frac{P_t^*}{P_s}\right)^{-\theta} Y_s$ is period s output of a firm that last set its price in period t, $Y_s = \left(\int_0^1 Y_s(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$ and $MC_s^t = \frac{W_s}{\alpha} A_s^{-\frac{1}{\alpha}} {Y_s^t}^{\frac{1-\alpha}{\alpha}}$ is the period s marginal cost of a firm that last set its price in period t. Furthermore,

$$\begin{aligned}
E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_t^*}{P_s}\right)^{-\theta} Y_s^{1-\sigma}\left(\frac{P_t^*}{P_s} - \omega\frac{MC_s^t}{P_s}\right)\right] &= 0 \\
\Leftrightarrow E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_t^*}{P_s}\right)^{1-\theta} Y_s^{1-\sigma}\right] &= E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_t^*}{P_s}\right)^{-\theta} Y_s^{1-\sigma}\omega\frac{MC_s^t}{P_s}\right] \\
\Leftrightarrow E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_t}{P_s}\right)^{1-\theta} Y_s^{1-\sigma}\right] &= \frac{P_t}{P_t^*}\omega E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_t}{P_s}\right)^{-\theta} Y_s^{1-\sigma}\frac{MC_s^t}{P_s}\right]
\end{aligned}$$

such that the price of re-optimizing firms is given by

$$\frac{P_t^*}{P_t} \;\; = \;\; \omega\frac{E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_s}{P_t}\right)^{\theta} Y_s^{1-\sigma}\frac{MC_s^t}{P_s}\right]}{E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\frac{P_s}{P_t}\right)^{\theta-1} Y_s^{1-\sigma}\right]} \tag{10}$$

The New Keynesian Phillips curve Defining $p_t^* = \frac{P_t^*}{P_t}$ and $mc_s^t = \frac{MC_s^t}{P_s}$, we can rewrite (10) as

$$p_t^* \;\; = \;\; \frac{\omega}{1+\pi_t}\frac{E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\Pi_{j=0}^{s-t}(1+\pi_{t+j})\right)^{\theta} Y_s^{1-\sigma} mc_s^t\right]}{E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\Pi_{j=0}^{s-t}(1+\pi_{t+j})\right)^{\theta-1} Y_s^{1-\sigma}\right]}$$

We will loglinearize this expression around a steady with has *zero inflation* (with $\mu = 0$), such that $\bar{p}^* = 1$ (see Woodford (2003) for a discussion, footnote 32 on p.179). Log-linearizing the numerator on the right hand side yields

$$(1-\beta\gamma)E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left((1-\sigma)\hat{y}_s + \widehat{mc}_s^t + \sum_{j=0}^{s-t}\theta\hat{\pi}_{t+j}\right)\right]$$

Log-linearizing the denominator on the right hand side yields

$$(1-\beta\gamma)E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left((1-\sigma)\hat{y}_s+\sum_{j=0}^{s-t}(\theta-1)\hat{\pi}_{t+j}\right)\right]$$

Therefore, the loglinear version of (10) is

$$\begin{aligned}\hat{p}_t^*+\hat{\pi}_t &= (1-\beta\gamma)E_t\left[\sum_{s=t}^{\infty}(\beta\gamma)^{s-t}\left(\widehat{mc}_s^t+\sum_{j=0}^{s-t}\hat{\pi}_{t+j}\right)\right]\\ &= (1-\beta\gamma)\left(\widehat{mc}_t^t+\hat{\pi}_t\right)+\beta\gamma\left(\hat{\pi}_t+(1-\beta\gamma)E_t\left[\sum_{s=t+1}^{\infty}(\beta\gamma)^{s-t-1}\left(\widehat{mc}_s^t+\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right)\right]\right)\end{aligned} \tag{11}$$

Recall that mc_s^t is the period s real marginal cost of a firm that re-optimizes in period t:

$$\begin{aligned}{mc^t}_s &= \frac{w_s}{\alpha}A_s^{-\frac{1}{\alpha}}{Y_s^t}^{\frac{1-\alpha}{\alpha}}\\ &= \frac{w_s}{\alpha}A_s^{-\frac{1}{\alpha}}\left(\frac{P_t^*}{P_s}\right)^{-\theta\frac{1-\alpha}{\alpha}}Y_s^{\frac{1-\alpha}{\alpha}}\\ &= mc_s(p_t^*)^{-\theta\frac{1-\alpha}{\alpha}}\left(\frac{\Pi_{j=0}^{s-t}(1+\pi_{t+j})}{1+\pi_t}\right)^{\theta\frac{1-\alpha}{\alpha}}\end{aligned}$$

where $mc_s=\frac{w_s}{\alpha}A_s^{-\frac{1}{\alpha}}Y_s^{\frac{1-\alpha}{\alpha}}$ is the "average" real marginal cost in period s. Loglinearizing this expression yields

$$\widehat{mc}_s^t = \widehat{mc}_s-\theta\frac{1-\alpha}{\alpha}\hat{p}_t^*+\theta\frac{1-\alpha}{\alpha}\left(\sum_{j=0}^{s-t}\hat{\pi}_{t+j}\right)-\theta\frac{1-\alpha}{\alpha}\hat{\pi}_t \tag{12}$$

Substituting (12) in (11) yields

$$\begin{aligned}\hat{p}_t^*+\hat{\pi}_t &= (1-\beta\gamma)\left(\widehat{mc}_t-\theta\frac{1-\alpha}{\alpha}\hat{p}_t^*+\hat{\pi}_t\right)+\beta\gamma\hat{\pi}_t\\ &\quad+\beta\gamma(1-\beta\gamma)E_t\left[\sum_{s=t+1}^{\infty}(\beta\gamma)^{s-t-1}\left(\widehat{mc}_s-\theta\frac{1-\alpha}{\alpha}\hat{p}_t^*+\left(1+\theta\frac{1-\alpha}{\alpha}\right)\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right)\right]\\ \left(1+\theta\frac{1-\alpha}{\alpha}\right)\hat{p}_t^* &= (1-\beta\gamma)\widehat{mc}_t+\beta\gamma(1-\beta\gamma)E_t\left[\sum_{s=t+1}^{\infty}(\beta\gamma)^{s-t-1}\left(\widehat{mc}_s+\left(1+\theta\frac{1-\alpha}{\alpha}\right)\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right)\right]\\ \left(1+\theta\frac{1-\alpha}{\alpha}\right)\hat{p}_t^* &= (1-\beta\gamma)\widehat{mc}_t+\beta\gamma\left(1+\theta\frac{1-\alpha}{\alpha}\right)E_t\left[\hat{p}_{t+1}^*+\hat{\pi}_{t+1}\right]\end{aligned} \tag{13}$$

To understand the last step, verify that

$$\begin{aligned}
\widehat{mc}_s^{t+1} &= \widehat{mc}_s - \theta\frac{1-\alpha}{\alpha}\hat{p}^*_{t+1} + \theta\frac{1-\alpha}{\alpha}\left(\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right) - \theta\frac{1-\alpha}{\alpha}\hat{\pi}_{t+1} \\
\Leftrightarrow \widehat{mc}_s^{t+1} + \sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j} &= \widehat{mc}_s - \theta\frac{1-\alpha}{\alpha}\hat{p}^*_{t+1} + \left(1+\theta\frac{1-\alpha}{\alpha}\right)\left(\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right) - \theta\frac{1-\alpha}{\alpha}\hat{\pi}_{t+1} \\
\Leftrightarrow \widehat{mc}_s + \left(1+\theta\frac{1-\alpha}{\alpha}\right)\left(\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right) &= \widehat{mc}_s^{t+1} + \sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j} + \theta\frac{1-\alpha}{\alpha}\left(\hat{p}^*_{t+1} + \hat{\pi}_{t+1}\right)
\end{aligned}$$

and that

$$\begin{aligned}
&\beta\gamma(1-\beta\gamma)E_t\left[\sum_{s=t+1}^{\infty}(\beta\gamma)^{s-t-1}\left(\widehat{mc}_s + \left(1+\theta\frac{1-\alpha}{\alpha}\right)\sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j}\right)\right] \\
&= \beta\gamma(1-\beta\gamma)E_t\left[\sum_{s=t+1}^{\infty}(\beta\gamma)^{s-t-1}\left(\widehat{mc}_s^{t+1} + \sum_{j=0}^{s-t-1}\hat{\pi}_{t+1+j} + \theta\frac{1-\alpha}{\alpha}\left(\hat{p}^*_{t+1} + \hat{\pi}_{t+1}\right)\right)\right] \\
&= \beta\gamma\left(1+\theta\frac{1-\alpha}{\alpha}\right)E_t\left[\hat{p}^*_{t+1} + \hat{\pi}_{t+1}\right]
\end{aligned}$$

Next reconsider equation (9):

$$\begin{aligned}
P_t &= \left((1-\gamma)(P^*_t)^{1-\theta} + \gamma P_{t-1}^{1-\theta}\right)^{\frac{1}{1-\theta}} \\
\Leftrightarrow 1 &= (1-\gamma)(p^*_t)^{1-\theta} + \gamma(1+\pi_t)^{\theta-1}
\end{aligned}$$

Loglinearizing this last expression yields

$$\hat{p}^*_t = \frac{\gamma}{1-\gamma}\hat{\pi}_t \tag{14}$$

Substituting (14) into (13) yields an expression for the evolution of aggregate inflation known as the *New Keynesian Phillips curve*:

$$\hat{\pi}_t = \kappa\widehat{mc}_t + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{15}$$

where $\kappa = \frac{(1-\beta\gamma)(1-\gamma)}{\gamma\left(1+\theta\frac{1-\alpha}{\alpha}\right)} > 0$, $0 < \beta < 1$. In contrast to more traditional Phillips curves, the New Keynesian Phillips curve implies that the (average) real marginal cost is the correct driving variable for the inflation process and that inflation is forward looking, with current inflation a function of expected future inflation. When a firm sets a price in the Calvo framework, it is concerned about the future evolution of prices as it is unable to alter its own price for a number of periods. Solving (15) forward, it is clear that inflation equals the present discounted value of current and future real marginal costs:

$$\hat{\pi}_t = \kappa E_t \sum_{s=t}^{\infty} \beta^{s-t} \widehat{mc}_s$$

The parameter κ measures the impact of period t real marginal cost on period t inflation and is a function of the structural parameters β, the discount factor, γ, the fraction of firms that cannot change their prices, θ, the elasticity of substitution among consumption goods and α, the curvature of the production function. Higher β lowers κ and means that the firm gives more weight to future expected profits. More nominal price rigidity (higher γ) reduces the sensitivity of current inflation to current marginal cost. The parameter κ is also decreasing in the curvature of the production function as measured by α and in the elasticity of demand θ. The larger α and θ, the more sensitive is the marginal cost of an individual firm to deviations of its price from the average price level: everything else equal, a smaller adjustment in price is desireable in order to offset expected movements in average marginal costs.

Aggregate Supply From the definition of average real marginal cost and the household's labor supply condition, we have

$$mc_t = \frac{\phi_n N_t^{\xi}}{Y_t^{-\sigma}} \frac{N_t}{\alpha Y_t}$$

Recall that Y_t was defined as

$$\begin{aligned} Y_t &= \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ &= A_t \left(\int_0^1 N_t(i)^{\alpha \frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \end{aligned}$$

As a first order approximation, we have

$$\hat{y}_t = a_t + \alpha \hat{n}_t$$

Using this insight we can write

$$\begin{aligned}\widehat{mc}_t &= (1+\xi)\hat{n}_t + (\sigma-1)\hat{y}_t \\ &= -\frac{1+\xi}{\alpha}\hat{a}_t + \left(\frac{1+\xi}{\alpha} + \sigma - 1\right)\hat{y}_t \\ &= \frac{1+\xi+\alpha(\sigma-1)}{\alpha}\left(\hat{y}_t - \hat{y}_t^n\right)\end{aligned}$$

such that the aggregate supply equation in the Calvo model becomes

$$\hat{\pi}_t = \kappa\frac{1+\xi+\alpha(\sigma-1)}{\alpha}\left(\hat{y}_t - \hat{y}_t^n\right) + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{16}$$

Model Analysis Consider the same parametrization of the model as before: $\alpha = 0.58$, $\beta = 0.988$, $\sigma = 1$, $\xi = 1$, $\chi = 10$ and $\mu = 0$. Now we have also have to choose a value for θ since it will affect the slope of the aggregate supply curve. In addition, there is the parameter γ measuring the degree if price rigidity. Gali, Gertler and Lopez-Salido (2001), set $\omega = 1.10$ (or $\theta = 11$) and estimate a Phillips curve as in (15) using quarterly US data to obtain a value of $\gamma = 0.475$. This estimate implies that the average duration of price rigidity $\frac{1}{1-\gamma}$ is approximately two quarters.

Figure 5 illustrates the impact of a money growth and technology shock for the case where both the shocks are i.i.d. It is evident that with staggered price setting inflation responds sluggishly to economic shocks. In the case of a money growth shock, the initial impact on prices is less proportional to the increase in the money supply, since only a fraction of goods prices are adjusted. In subsequent periods, as more and more firms reset prices, the LM-curve and AD curve return to their initial positions, but only gradually so. The persistent output increase and slow response of inflation, together with the decrease in the nominal interest rate after a monetary expansion are now more consistent with the evidence from the structural VAR literature.

In contrast to the model with one period pre-set prices, output now increases after a positive technology shock, albeit much less so then in the flexible price model. This is because now a fraction of the firms can change prices in order to benefit from the technology improvement.

Figure 6 displays the responses to persistent money growth and technology shocks: as before, $\rho_\theta = 0.48$ and $\rho_a = 0.9$. After a persistent money growth shock, the effect of higher anticipated future inflation further stimulates aggregate demand and the output increase is now larger. In response to a persistent technology shock, the combination of aggregate supply effects and aggregate demand effects (through inflation, expected future inflation

Figure 5: Calvo Sticky Price Model: i.i.d. shocks

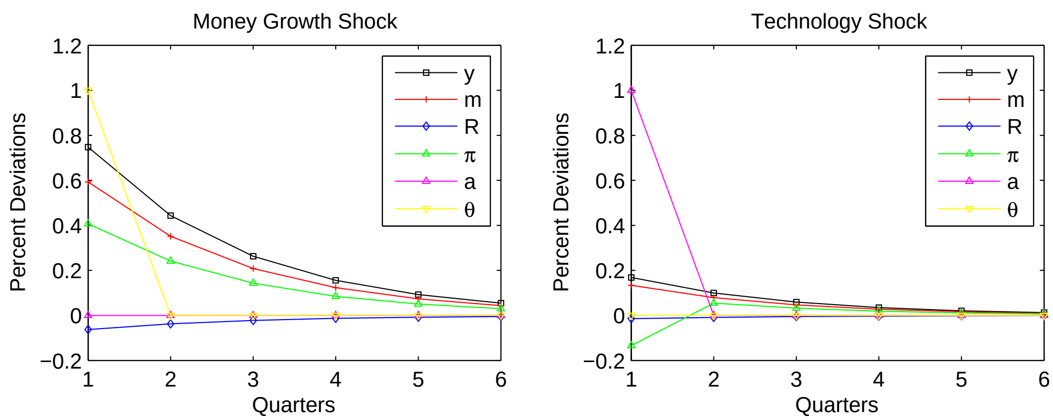
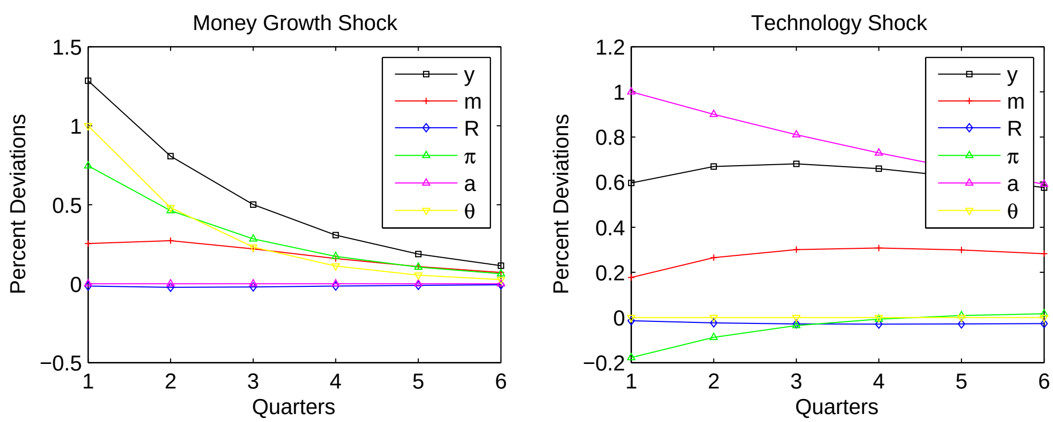


Figure 6: Calvo Sticky Price Model: persistent shocks



and expected future output) generates a hump-shaped response of output.

The Matlab-program used to compute these impulse responses is **stickyprice3.m**.

4 A Basic Model of the Effects of Monetary Policy

This section introduces a more realistic formulation of monetary policy into the Calvo staggered pricing model of the previous section. Recall that up until now, we have modeled monetary policy in a very simplified manner by positing a random stochastic process for the growth rate of the money supply. In this section, we will think of the government, or central bank, as implementing monetary policy through control of the nominal interest rate.

Taylor Rule Monetary policy might be specified by *a Taylor rule* of the form

$$\frac{1+R_t}{1+\bar{R}} = \Psi\left(\frac{1+R_{t-1}}{1+\bar{R}}, \frac{1+\pi_t}{1+\pi^*}, \frac{Y_t}{Y_t^n}, e_t^{\epsilon^i}\right)$$

where $\Psi(1,1,1,1) = 1$, π^* is a target inflation rate and ϵ_t^R is an exogenous mean zero disturbance to the monetary rule. A rule of this form was first introduced by Taylor (1993) and has proven to be a reasonable empirical description of the behavior of many central banks. In loglinear form we may write the Taylor rule as

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\rho_\pi \hat{\pi}_t + \rho_y \hat{x}_t\right) + \epsilon_t^R \tag{17a}$$

where $0 \leq \rho_R < 1, \rho_\pi, \rho_y \geq 0$. The parameter ρ_R captures policy inertia that may be due to the policy makers' preference for interest rate smoothing. The coefficients ρ_π, resp. ρ_y measure the policymakers' response to deviations of inflation from the target (steady state) level, resp. movements in the output gap. Combined with the IS and AS (or Phillips) relationship

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - E_t \hat{\pi}_{t+1}\right) + u_t \tag{17b}$$

$$\hat{\pi}_t = \kappa\lambda \hat{x}_t + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{17c}$$

where $\lambda = \frac{1+\xi+\alpha(\sigma-1)}{\alpha}$, $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$ is the output gap and $u_t = E_t y_{t+1}^n - y_t^n$, we obtain a closed system of equations that describe the dynamics of inflation, the nominal interest rate and the output gap that is useful to study the transmission of monetary policy. Note that u_t depends only on the supply shock.

Taylor Principle The following discussion is based on Bullard and Mitra (2002). Let's consider a simplified version of the Taylor in which $\rho_i = 0$. Combining equations (17a) through (17c), we can eliminate $\hat{R}_t$ and obtain

$$\begin{bmatrix} -1-\frac{\rho_y}{\sigma} & -\frac{\rho_\pi}{\sigma} \\ -\kappa\lambda & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} E_t \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} -1 & \frac{1}{\sigma} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_t \\ \epsilon_t^R \end{bmatrix}$$

or

$$\begin{aligned}
\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} &= \begin{bmatrix} -1-\frac{\rho_y}{\sigma} & -\frac{\rho_\pi}{\sigma} \\ -\kappa\lambda & 1 \end{bmatrix}^{-1}\begin{bmatrix} -1 & -\frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} E_t \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} -1-\frac{\rho_y}{\sigma} & -\frac{\rho_\pi}{\sigma} \\ -\kappa\lambda & 1 \end{bmatrix}^{-1}\begin{bmatrix} -1 & \frac{1}{\sigma} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_t \\ \epsilon_t^R \end{bmatrix} \\
&= \frac{1}{\sigma+\rho_y+\rho_\pi\kappa\lambda}\begin{bmatrix} \sigma & 1-\beta\rho_\pi \\ \sigma\kappa\lambda & \kappa\lambda+\beta(\sigma+\rho_y) \end{bmatrix} E_t \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + \frac{1}{\sigma+\rho_y+\rho_\pi\kappa\lambda}\begin{bmatrix} \sigma & -1 \\ \sigma\kappa\lambda & -\kappa\lambda \end{bmatrix}\begin{bmatrix} u_t \\ \epsilon_t^R \end{bmatrix} \\
&= M^1 E_t \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + M^2 \begin{bmatrix} u_t \\ \epsilon_t^R \end{bmatrix}
\end{aligned}$$

The system will have a unique stationary solution only if M_1 has both eigenvalues inside the unit circle. The characteristic polynomial of M_1 is

$$\Lambda^2 - (m_{11}+m_{22})\Lambda + (m_{11}m_{22} - m_{12}m_{21}) = 0$$

where m_{ij} is the ij-th element of M_1. It can be shown the stability conditions amount to

$$\begin{aligned}
\mid m_{11}m_{22} - m_{12}m_{21} \mid &< 1 \\
\mid m_{11} + m_{22} \mid &< 1 + m_{11}m_{22} - m_{12}m_{21}
\end{aligned}$$

Verify that $m_{11}m_{22} - m_{12}m_{21} = \frac{\beta\sigma}{\sigma+\rho_y+\rho_\pi\kappa\lambda}$ such that the first condition reduces to

$$-(1-\beta)\sigma < \rho_y + \rho_\pi\kappa\lambda$$

which is always satisfied since $0 < \beta < 1$. The second condition can be written as

$$\kappa(\rho_\pi - 1) + (1-\beta)\rho_y > 0$$

This last condition will not be satisfied for any value of ρ_y and ρ_π. We should therefore be very careful when introducing exogenous policy rules, as they might not yield a unique equilibrium and may be consistent with multiple equilibria. Suppose for instance that

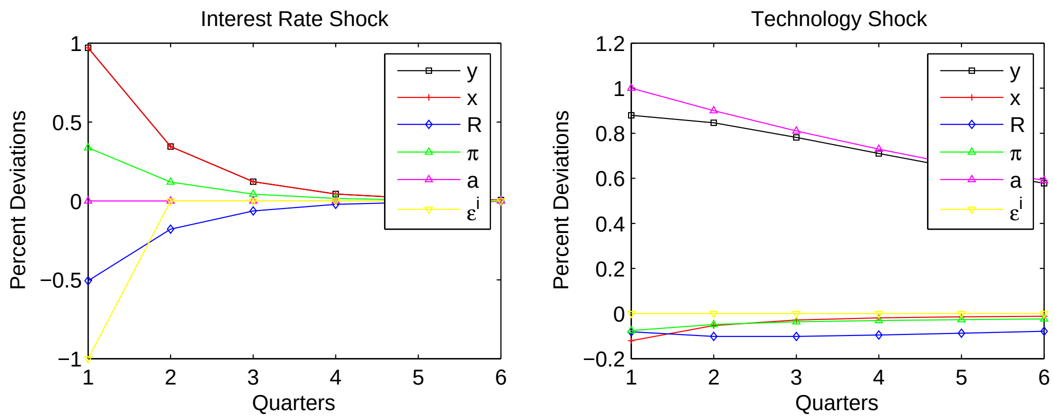
$\rho_y = 0$, such that condition (18) becomes

$$\rho_\pi > 1$$

The practice of setting $\rho_\pi > 1$ is know as the *Taylor principle*: it prescribes that the nominal interest rate is to react more than proportional to the rate of inflation.

Model Analysis With a more realistic formulation of monetary policy in the form of a Taylor rule, we can now turn to a numerical evaluation of the new model and compare its performance to the structural VAR evidence on monetary shocks. First note that we no longer need to specify a value for χ (governing the interest rate elasticity of money demand) as long as we are only interested in the dynamics of inflation, output and the nominal interest rate. The money supply is now endogenous and is adjusted by the central bank in order to achieve the desired nominal interest rate. However, we must chose values for the parameters of the Taylor rule. Following Woodford (2003), let's consider $\rho_i = 0.7$, $\rho_\pi = 2$ and $\rho_y = 1$ as a realistic description of monetary policy. The technology shock and the interest rate shock are both AR(1) processes with persistence 0.9 and 0 respectively. The values of all remaining parameters are the same as before.
Figure 7 plots the impulse responses to a negative innovation in the nominal interest rate (a "federal funds rate shock") and a persistent technology shock for the Calvo model with the Taylor rule. The response to an exogenous interest rate decrease is similar the structural VAR evidence along several dimensions: output increases persistently and so does inflation. The key monetary transmission mechanism operates through changes in the real interest rate which affect consumption (the *interest rate channel*). In contrast to the flexible price economy, the central bank can affect the real interest rate (i.e. shift the LM curve) by changing the nominal rate because of the sluggish response of inflation. The decrease in the real interest rate causes the households to increase current consumption (i.e. a move along the IS curve), which stimulates aggregate demand. There remain however some important discrepancies between the model and the empirical evidence: for instance, there is no hump-shaped response of output as in the data and there was a long period of inflation inertia in the data, but not in the model.
Note that in response to a positive technology shock, the central bank reacts to the decrease in inflation ($\hat{\pi}_t < 0$) as well as the widening of the ouput gap ($\hat{x}_t < 0$) by lowering nominal interest rates. The technology shock raises the natural output level (or potential output) and the central bank reacts by accommodating. The resulting decrease in real interest rates

Figure 7: Calvo Model with Taylor Rule



stimulates aggregate demand and helps closing the output gap.

The Matlab-program used to compute these impulse responses is **stickyprice4.m**.