

Not-For-Publication Appendix:
A Reconciliation of SVAR and Narrative
Estimates of Tax Multipliers*

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In the paper, we mainly consider the VAR specifications of relatively small dimension analyzed by Blanchard and Perotti (2002). In this web appendix, we discuss further results for VAR systems of larger dimensions. In particular, we add variables such as government debt, monetary policy instruments and inflation, as well as variables that may contain additional information on expectations of fiscal policy.

We show that

1. Estimating impulse responses for any number of n variables requires no identifying assumptions beyond exogeneity of the proxy m_t .
2. Elasticities in Blanchard-Perotti tax revenue equations with an arbitrary number of endogenous variables can be estimated as long as government spending does not respond contemporaneously to any of the additional variables.
3. The tax multipliers estimated in larger VARs are very similar to those of the original smaller specification.
4. Estimates of the output elasticity of tax revenues remain high in larger VAR systems.

1 Identification in Larger VAR Systems

To organize our discussion of the larger VARs, consider now the following parametrization of the relationship between the VAR residuals u_t structural shocks ε_t :

$$\begin{aligned} u_t^T &= \tilde{\theta}_X u_t^X + \tilde{\sigma}_T \varepsilon_t^T, \\ u_t^X &= \tilde{\zeta}_T u_t^T + \Sigma_X \varepsilon_t^X, \end{aligned} \tag{A-1}$$

where u_t^T is the reduced form VAR residual associated with the tax revenues equation, whereas u_t^X contains the reduced form residuals associated with an arbitrary number n of additional variables. $\tilde{\theta}_X$ is $1 \times n$, $\tilde{\sigma}^T$ is a scalar, $\tilde{\zeta}_T$ is $n \times 1$ and Σ_X is $n \times n$, ε_t^T is the structural tax shock, whereas ε_t^X contains all other structural shocks. The identifying assumptions now become

$$E[m_t \varepsilon_t^T] = \phi \neq 0, \quad (\text{A-2})$$

$$E[m_t \varepsilon_t^X] = 0, \quad (\text{A-3})$$

We first establish that estimating impulse responses for any number of n variables requires no identifying assumptions beyond exogeneity of the proxy m_t . This can be easily seen by considering the following implementation of the identifying restrictions in (A-2)-(A-3) using IV regressions:

1. Regress u_t^X on u_t^T using m_t as instruments, which yields an unbiased estimate of $\tilde{\zeta}_T$. Define the residual v_t^X .
2. Regress u_t^T on u_t^X using v_t^X as instruments, which yields an unbiased of $\tilde{\theta}_X$. Define the residual v_t^T .
3. Estimate $\tilde{\sigma}^T$ from the variance of v_t^T .

The contemporaneous impact vector β_T associated with tax shocks is given by

$$\beta_T = \begin{bmatrix} I + \tilde{\theta}_X (I - \tilde{\zeta}_T \tilde{\theta}_X)^{-1} \tilde{\zeta}_T \\ (I - \tilde{\zeta}_T \tilde{\theta}_X)^{-1} \tilde{\zeta}_T \end{bmatrix} \tilde{\sigma}^T.$$

which is only a function of $\tilde{\zeta}_T$, $\tilde{\theta}_X$ and $\tilde{\sigma}^T$. Therefore, the assumptions on the proxy in (A-2)-(A-3) suffice to identify the column of the impact matrix \mathcal{B} and no additional identifying assumptions are needed to estimate impulse response functions to tax shocks (or the reliability of the proxy). In the paper, the only reason to impose the additional restriction that government spending does

not respond contemporaneously to output $\gamma_Y = 0$ is that this is necessary to identify the parameters $(\theta_Y, \theta_G, \zeta_T \zeta_G, \dots)$ of the Blanchard-Perotti system in equation (3) of the main paper.

Key to the comparison with the Blanchard-Perotti identification strategy are the parameters of the tax revenue equation. Consider the Blanchard-Perotti tax revenue equation for the case of an arbitrary number of endogenous variables

$$u_t^T = \theta_G \sigma_G \varepsilon_t^G + \theta_{X_1} u_t^{X_1} + \theta_{X_2} u_t^{X_2} + \dots + \sigma_T \varepsilon_t^T \quad (\text{A-4})$$

and contrast with the tax revenue equation in (A-1) repeated here for clarity:

$$u_t^T = \tilde{\theta}_G u_t^G + \tilde{\theta}_{X_1} u_t^{X_1} + \tilde{\theta}_{X_2} u_t^{X_2} + \dots + \tilde{\sigma}_T \varepsilon_t^T \quad (\text{A-5})$$

If we assume that government spending does not respond contemporaneously to any nonfiscal shock and $u_t^G = \sigma_G \varepsilon_t^G + \gamma_T \varepsilon_t^T$, it follows that $\tilde{\theta}_G = \theta_G$, $\tilde{\theta}_{X_1} = \theta_{X_1}$, $\tilde{\theta}_{X_2} = \theta_{X_2}, \dots$. Hence, the coefficients of the Blanchard-Perotti tax revenue equation with an arbitrary number of endogenous variables can be obtained by the proxy SVAR estimate of $\tilde{\theta}_X$ described above as long as we assume that government spending does not respond contemporaneously to any of the additional variables. The latter assumption is also made by typical larger dimensional applications of the Blanchard Perotti identification strategy in the literature. However these applications need to assume in addition that γ_T and $\theta_{X_1}, \theta_{X_2}, \dots$ are known. Finally, note that (A-4) and (A-5) are very similar such that in practice the elasticities $\tilde{\theta}_{X_1}, \dots$ are likely to be very close to θ_{X_1}, \dots even if government spending does to some extent respond contemporaneously to nonfiscal shocks.

2 Estimation Results

We consider several augmented versions of the benchmark VAR specification that included tax revenues, government spending (G) and output (Y). First we add a measure of federal debt held by the public (DEBT). Next, we run a typical monetary VAR specification that includes DEBT as well as the federal funds rate (FF), the PCE price level (P) and Nonborrowed Reserves (NBR).¹ Finally, we add to all of these variables, in turn, the ‘fiscal foresight’ variables discussed in the main paper: the defense dummy (DEFD) of Ramey (2011), the excess returns series (EXCR) of Fisher and Peters (2010) and finally the implicit expected tax rate based on municipal bond spreads (MBS, 1 year maturity) of Leeper, Walker and Yang (2011). All specifications have the same deterministic terms, lag length, and sample size as the benchmark specifications, except for the specification with the MBS variable which is only available from 1953Q2 onwards.

The upper left panel of Figure A-1 depicts the output response and confidence intervals for the benchmark proxy SVAR with only government spending and output as additional variables together with the output responses of all larger specifications. The remaining panels show the output responses and confidence intervals of the larger systems individually. The point estimates are in all cases very similar to the benchmark specification. As is to be expected, the confidence intervals become somewhat wider when more variables are added, but in all cases the output response is significant for the first 9 quarters after the shock. We conclude that including the additional variables has very little effect on our results.

¹DEBT is Federal Debt Held by the Public from Favero and Giavazzi (2012) (*DEBTHP*), divided by the GDP deflator and population. P is the implicit deflator for Personal Consumption Expenditures (NIPA Table 1.1.9 line 2); FF is from Romer and Romer (2010) which they extended back to 1950Q1; NBR is from FRED (series BOGNONBR), extended back to 1950Q1 by subtracting borrowed reserves (FRED: BORROW) from total reserve balances (FRED: RESBALNS) after adjusting for changes in reserve requirements using the reserve adjustment magnitude from the St. Louis Fed. All variables except FF are in logs. The other variables are described in the main paper.

Table A-1 shows a selection of the elasticity estimates of the tax revenue equation associated with every VAR specification.² A few results stand out. First, the output elasticity of tax revenues as estimated by $\tilde{\theta}_Y$ remains very high in all of the larger specifications, with point estimates ranging from 2.55 to 2.77. When other highly procyclical variables, such as nominal interest rates, are included, it is not surprising that we obtain somewhat lower values relative the specification that only includes output. Thus we view these results as consistent with the high cyclical sensitivity of tax revenues estimated in the simpler specifications. Moreover, Figure A-1 shows that the tax multipliers are almost unaffected by these lower output elasticities because the multipliers now also depend on the sensitivity to the other endogenous variables. Second, one coefficient that is both economically and statistically strongly significant in all specifications is the positive elasticity of tax revenues to government debt. This is suggestive for an important feedback channel from the level of government debt to tax revenues that is consistent with the tendency for debt stabilization as documented for instance by Bohn (1998) and Corsetti, Meier and Mueller (2012) and for ‘passive’ fiscal policy in the terminology of Leeper (1990). Third, the coefficient on the municipal bond spread series is significantly positive, which is suggestive for the informational content of this variable for tax rates. The remaining coefficients are mostly insignificant. Finally, the estimate of the reliability of the narrative proxy is very similar across all specifications.

References

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²Some of the coefficients are not shown for brevity, but none of these were economically or statistically significant.

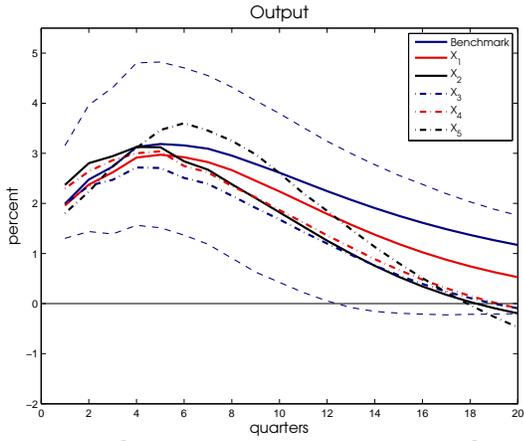
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Table A-1 Elasticities in Larger VAR Systems

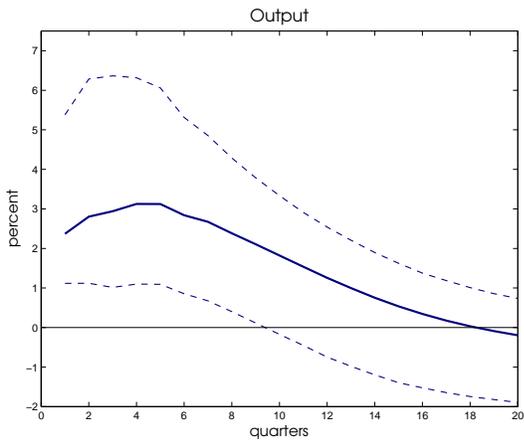
Additional Variables X	$\tilde{\theta}_Y$	$\tilde{\theta}_G$	$\tilde{\theta}_{DEBT}$	$\tilde{\theta}_{FF}$	$\tilde{\theta}_P$	$\tilde{\theta}_{MBS}$	Reliability
$[GDP, G]$ (Benchmark)	3.13 [2.73, 3.55]	-0.20 [-0.35, -0.07]					0.57 [0.50, 0.61]
$[GDP, G, DEBT]$	2.71 [2.37, 3.10]	-0.15 [-0.29, -0.04]	0.52 [0.27, 0.69]				0.58 [0.52, 0.62]
$[GDP, G, DEBT, FF, P, NBR]$	2.55 [1.51, 3.72]	-0.16 [-0.69, 0.42]	0.57 [0.23, 0.85]	0.73 [-0.01, 1.60]	-0.04 [-0.19, 0.08]		0.55 [0.44, 0.60]
$[GDP, G, DEBT, FF, P, NBR, DEFD]$	2.63 [1.33, 4.08]	-0.14 [-0.77, 0.56]	0.49 [0.05, 0.80]	0.48 [-0.45, 1.59]	-0.04 [-0.22, 0.13]		0.48 [0.33, 0.56]
$[GDP, G, DEBT, FF, P, NBR, EXCR]$	2.74 [1.62, 4.21]	-0.06 [-0.69, 0.61]	0.52 [0.13, 0.82]	0.27 [-0.71, 1.24]	-0.02 [-0.19, 0.11]		0.53 [0.38, 0.60]
$[GDP, G, DEBT, FF, P, NBR, MBS]$	2.77 [1.79, 4.28]	0.26 [-0.34, 1.05]	0.55 [0.16, 0.92]	0.42 [-0.55, 1.47]	-0.09 [-0.28, 0.06]	0.15 [0.02, 0.30]	0.57 [0.42, 0.63]

Values in parenthesis are 95% percentiles computed using 10,000 bootstrap replications.

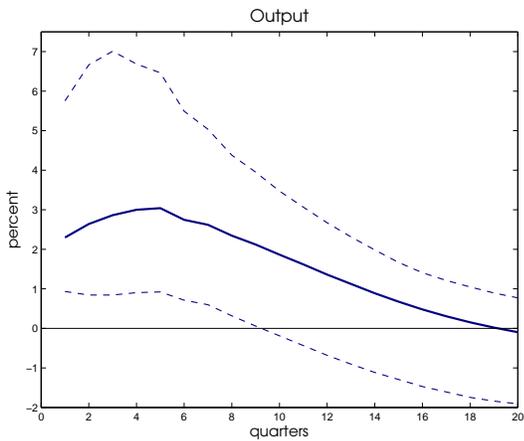
Benchmark and Larger Systems



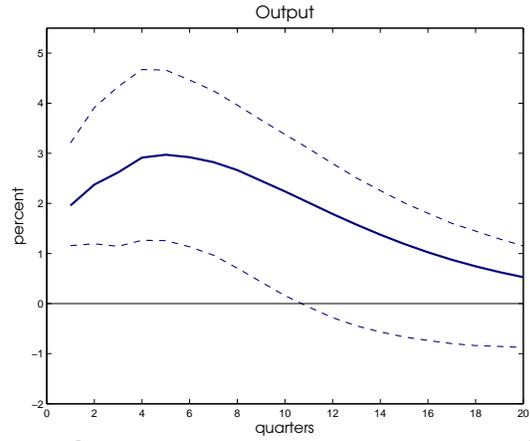
$$X_2 = [GDP, G, DEBT, FF, P, NBR]$$



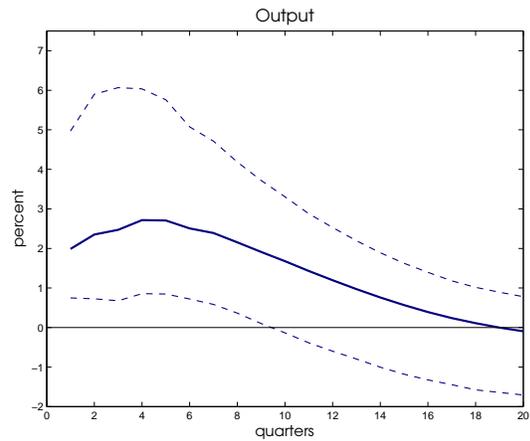
$$X_4 = [GDP, G, DEBT, FF, P, NBR, EXCR]$$



$$X_1 = [GDP, G, DEBT]$$



$$X_3 = [GDP, G, DEBT, FF, P, NBR, DEF D]$$



$$X_5 = [GDP, G, DEBT, FF, P, NBR, MBS]$$

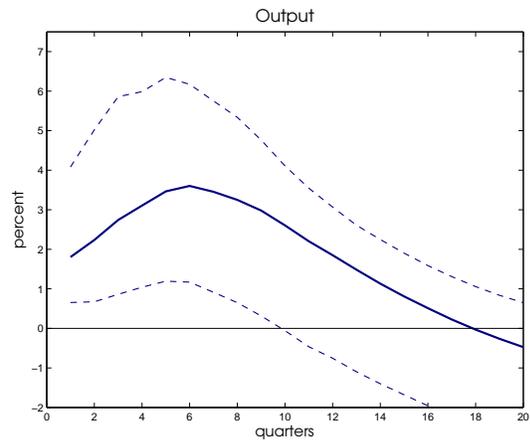


Figure A-1 Larger VAR Systems: Output Response to a Tax Cut of 1% of GDP. Broken lines are 95% bootstrapped percentiles.